Preaching or Practising?

Action Research into the Teaching of Early Childhood Mathematics Education

Rachel Deitcher

DPhil
Submitted to the University of Sussex
April, 2003
[2004]

Acknowledgements

I would like to celebrate the completion of this thesis by acknowledging the help and support of family, friends and colleagues who have, each in their own way, contributed to its completion.

My thanks go to Ada Svidovsky, longtime friend and professional partner, whose conversation has enriched me and whose encouragement has sustained me through difficult times.

To Jack Whitehead, whose generous help and enthusiasm at a critical juncture were instrumental in enabling me to complete this thesis.

To Nita Schechet for her unfailing help and support. Her ability to appreciate and value the ideas of others on their own terms distinguishes her both as an educator and as a friend.

To my parents for the love and support they have shown me over the years.

To my husband, Avraham, and my children Yonatan and Shelli, for standing by me in good times and bad. This accomplishment is theirs as well as my own.

And finally, to my grandfather, Dr. Ezra Lozinski, for both believing in me and planting the seeds of my interest in learning. This thesis is dedicated to him.

University of Sussex

Rachel Deitcher DPhil

Preaching or Practising? Action Research into the Teaching of Early Childhood Mathematics Education

Summary

In this thesis I present a self-study action research project which looked into ways to improve my teaching of a college-based education course in the pedagogy of early childhood mathematics. My desire to change my practice stemmed from a feeling of dissatisfaction with my teaching of certain aspects of the course, and from the knowledge that, once they became teachers, many former students did not put what they had learned in the course into practice. Three major educational theories influenced the choice of teaching strategies incorporated into my work: constructivist theory, which is used to inform the teaching of mathematics pedagogy; situated learning theory, which focuses on the better use of the contexts in which the course is situated in order to increase its effectiveness; and the theory of Mediated Learning Experience which suggests ways of looking at interactions between teachers and students and of increasing the effectiveness of these interactions.

Chapter I describes the context in which the study took place. Chapter II reviews the literature on the development of expertise in professional practice and on the three theories that form the basis of my work. Chapter III deals with the methodology and methods used. The methodology section reviews the characteristics of self-study action research, the methodological and practical problems of self-study in one's own classroom and the challenge it poses to validity. The section on methods describes the methods used to collect and analyze the data. Chapter IV presents background knowledge regarding previous courses taken by the students involved in the study and regarding their perceptions and beliefs vis a vis mathematics and mathematics education. The following three chapters, Chapters V through VII, present accounts of three college-based modules which form the major part of the course Didactics of Early Childhood Mathematics Education which is the focus of this study. These accounts are based on tape recordings, post lesson notes, my reflective diary and student written responses. Chapter VIII is a critical review and evaluation of the outcomes of the course vis a vis the perceptions and beliefs of my students at the end of the year. It includes a review of the literature on developing beliefs in professional education. Chapter IX summarizes my learning and looks at the effects that this self-study project had on my teaching after its completion.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ACRONYMS

TIMSS (1985)	Third International Mathematics and Science Study
TIMSS (1999)	Trends in International Mathematics and Science Study
NCTM	National Council of Teachers of Mathematics
MLE	Mediated Learning Experience

Introduction

OK kids. Time for math. Take out your math books and start working on page 45. Any questions for now?

OK kids. Time for math. Sit down and I'll explain what you have to do. On page 35 of your books you'll see there is a table that you have to fill out . Can anyone tell me what you have to put in the first square? Right, Sari. And now Yonatan - do you know what has to go in the next square? Can anyone help him? Good for you, Rafi. Now, has everyone understood? No questions? Go to your places and work from page 35 to page 37. If there are any questions I'm here to help.

OK kids. Time for math. Take out your math books and start working. If anyone has finished the chapter on addition of two-digit numbers come and show me your books before you go on. If there are any questions I'm here.

OK kids. Time for math. Groups A, B and D - you will continue working in your books. Group C - I have a worksheet for you to practice the times table. This morning I want to sit with group E. I want to go over with you what you have to do in the chapter on multiplication before you go any further.

I have been involved in mathematics education in Israel for more than twenty years. During that time I have experienced mathematics education from a number of different angles - first as a general classroom teacher with a particular interest in mathematics education, later as a mathematics specialist at an elementary school in Jerusalem, and for the last ten years as a lecturer teaching the didactics of early childhood mathematics at David Yellin Teachers College. During my time at the college I have also acted as mathematics supervisor in three elementary schools in the city. At the time that I conducted this research study the above scenarios were typical of most mathematics classes in Israel, among them classes in schools where I supervised. Although in recent years pockets of alternative approaches to mathematics education have begun to develop in different parts of the country, this style of teaching is still widespread.

The common denominator of all of the scenarios is that children spend their time in mathematics classes working in a series of workbooks, beginning on page one in September and ending with the last page of the last workbook at the end of the year. The commonly accepted role of the teacher is to explain what has to be done in the book - and "understanding" by the children means that they know how to do what is necessary in order to complete each page. Assessment generally consists of the teacher checking the children's work and the occasional written test (often in anticipation of parent-teacher meetings). This test is presumed to give a more all-round picture of the child's mathematical knowledge to date. Taking into account the busy schedule, hectic

pace and classes of up to 40 children of typical elementary schools in the country, this minimalistic view of the teacher's role in their pupils' mathematical development is often well- suited to the perceived needs of many teachers.

> When we render the conventional as the useful / or rational /, it also becomes for us "natural", in the double sense of inherent in nature and normal in culture.
>
> (Sahlins, 1976 in Lave,1988, p. 92)

The expectation held by principals and often parents as well is that children complete the workbooks reasonably successfully and that they receive a passing grade in written tests. As such, teachers who are considered responsible vis-a-vis their mathematics programmes are those who pace the work accordingly. Less "successful" teachers are those whose pupils, when nearing the end of the year, are still far from having completed the workbooks. In that instance the last month or two of the school year are given to marathon mathematics sessions in order to do so. If, on the other hand, the "worst" should happen, i.e., the children have not completed the work by the end of the year, they are instructed to do so at home during the summer vacation. The teachers of these children can then console themselves that, from their point of view at least, they have done their best. This is done in the hope, however faint, that in this way the children will learn as much as possible of the mathematics that they will need in the following year.

The challenge and the opportunity of teacher education in Israel has been to effect change in the way mathematics is taught in our schools. Since beginning my work at the college, I have been struck by the opportunity I have been offered to reach hundreds of future teachers and introduce them to a view of mathematics education which encourages learning based on children's personal knowledge, experience, and interests. In order to best take advantage of this opportunity, however, my own teaching needed to be such that it would deeply affect my students' attitudes, perceptions, beliefs and understandings regarding mathematics and mathematics education.

A number of values that I held came into play here. The first was my commitment to the education and betterment of my students. This commitment reflects a belief in the propensity of human beings to learn, and thus develop and change, not only in what they know but also in their learning potential - in the actual intelligence with which they will approach problems of learning in the future (Feuerstein, 1999). As a teacher, it is my responsibility to teach in such a way that enables this development to occur. Because

the improvement of my own teaching can help make this possible, I am committed to make conscious attempts at improving my practice. In addition, I expect, and explicitly express the expectation, that my students do the same.

Two additional values, in which there seems to be a built-in contradiction, need to be mentioned here. On the one hand I believe that it is the right of all students to come to their own conclusions. I believe that t is not my role to tell them what they to believe. On the other hand, it is my belief that there is a certain kind of education, in which students are encouraged to construct their own understandings, that has the greatest potential to result in the deep and meaningful learning that will allow them to come to well-based conclusions, rather than retain unexamined beliefs based on previous experience. As such, I am interested in my students arriving at conclusions regarding education that are based on the literature that I deem important, and that are therefore somewhat similar to my own. Although it would seem that as a teacher of teachers a commitment to a professional view of education that can ensure teaching of the highest quality (Buchmann, 1986) is my major responsibility, the contradiction between my desire to allow them to come to their own conclusions and my desire for them to to choose the kind of education that I consider to be best was, and to some extent continues to be, a continual source of discomfort for me in my work.

From reactions of students at the college, and from echoes heard from teachers who had formerly been my students, I was aware of having attained a certain amount of success in introducing an alternative view of mathmeatics and mathematics education. Nonetheless, I was far from satisfied. Too often I would receive word of former students who, as pre-school teachers, had made only marginal use of their learning in the course, or, as first- or second-grade teachers, had succumbed to the mode of mathematics teaching characteristic of the schools in which they worked. In addition, I was keenly aware of the fact that I was unable to point to specific reasons that had caused a certain lesson that I taught to be particularly successful or unsuccessful. Having had an extremely successful and rewarding experience in my first year of college-level teaching, I can remember feeling nervous and unsure of my ability to achieve similar success a second time round.

In addition to this, was my awareness of the gap that existed between educational principles to which I adhered and the character of much of my actual teaching. While I believed strongly in the importance of learners actively building their own knowledge, I often found myself attempting to 'provide them with information' through lecturing. In

this instance, as with that of my inability to consciously repeat successful teaching practices, I felt that I did not have the tools which could allow me to look critically at what I had done: to analyze, understand and improve my practice. I needed these tools to provide me with a feeling that I had control over my work, to allow me to live my educational values more fully.

When I was first introduced to the idea of action research, in a lecture given by the director of our college in 1996, I was impressed by the potential that this research approach might have for helping me to develop those tools that I was lacking. A number of months later I joined a programme of studies at The University of Sussex which provided me with the opportunity to carry out an action research project. The goal of my project was to improve my functioning as a teacher of the didactics of early childhood mathematics in order to broaden and deepen the educational influence of the course. This would hopefully allow me to contribute more significantly to the reform of mathematics education in the country. Although my sights were always set on this wider goal, this was at base a self-study project (Whitehead, 2000): I was interested in looking at and developing my own practice.

In order to achieve these goals I first turned to three major educational theories, each of which offered ways of looking at education which I felt to be crucial for the improvement of my practice, and which therefore strongly influenced the choice of teaching strategies to be incorporated into my action research. These were:

* social constructivist theory as expounded and developed by researchers in the area of mathematics education, primarily Cobb & Yackel, (1996, 1998), Ball, (1988, 1990a), Simon (1995), and Carpenter & Fennema (1992).

* the theory of Mediated Learning Experience (Feuerstein, 1991, 1997)

* situated learning theory, as developed by Lave (1988, 1996), Brown et al. (1989) and Rogoff (1990).

The Contributions of this Research

Although some have seen the above theories as conflicting and therefore incompatible (Lave, 1988; Lerman, 1994), my understanding of their relevance and importance for my practical work compelled me to incorporate them both into my teaching while

attempting to integrate them theoretically. Eventually I began to understand that as a practising teacher I could not be locked into one complete, consistent and exclusive (excluding) theoretical framework which rejected others as being theoretically conflicting or inconsistent. Rather, the search for solutions to the difficult and diverse problems of teaching cries out for inclusiveness and integration, allowing the utilization of varied points of view to the benefit of multi-faceted human situations.

An additional factor contributing to the inadequacy of any one, or any combination of, existing theory for practical work is pointed to by Schon (1983) in his formulation of the concept of 'technical rationality', the term he uses for the applied science view of the relationship between theory and practice. In this Schon holds that research-based theoretical knowledge cannot be applied unproblematically to real-life practice, and cannot adequately account for the expertise that characterizes the practice of competent professionals.

The eventual outcome of my attempts to solve the practical and unique problems of my teaching has been the development of my own personal, practical theory related to effective teaching. This, my 'living educational theory' (Whitehead, 1993), incorporates research-based theories while going past them and relating specifically to my own understandings derived from and through my actual teaching. The theory thus developed, which continues to change and develop as it is constantly informed by new practical and theoretical knowledge, both guides my practice and allows me to gradually and continually reduce the gap between it and my principles. The way in which I have understood the necessity of theoretical inclusiveness in professional practice, as well as the reflexive view of the connection between theory and practice in practitioner research, may be seen to be contributions of this dissertation to both theory and practice in the field of practitioner research . In addition, the implications of my work may be seen at two additional levels: that relating to the ways in which a pre-service course in mathematics education can have greater educational influence on the personal theories of future teachers of early childhood mathematics and that pertaining to the theory and practice of teacher education in general.

At the first level, in anticipation of the explicit theoretical understanding described above, my attempts to increase the educational influence I had on my students' learning and on their development as teachers led me to take into account factors relating to the students as complete entities, as human beings whose cognitive, social, emotional and professional development are intimately interconnected one with the other. Taking

into account these facets of human makeup may be seen to be all the more crucial when dealing with the subject of mathematics as it is approached by the almost exclusively female population characteristic of early childhood education. The cognitive, social and emotional difficulties so often experienced by these students in their previous experience with mathematics, along with the cultural mores relating to the essence of mathematics and the ways in which it should be taught, often determine an attitude towards mathematics education that is narrow and inflexible, and far afield from contemporary professional understandings of good practice.

As in all things, teachers' theories of what mathematics and mathematics education are, are bound to affect their practice (Claxton, 1984). In my research I have clarified ways in which I can provide my students with the opportunity to experience mathematics and its teaching as relevant, interesting and important, both for themselves and for their future pupils. Simultaneously, I encouraged them to use their growing pedagogical understanding to look cortically at mathematics curricula as well as to reflect deeply and critically on their own work as teachers of mathematics. These opportunities for reflection allowed them to integrate the theoretical knowledge they were being introduced to, with their practice as student teachers of mathematics, thereby beginning to develop more informed personal theories (ibid.) regarding mathematics education .

At the second level, from the point of view of teacher education in general, this thesis is a contribution to the case literature in teacher education which will allow teachers to have access to richly developed portrayals of the practice of teacher education. Shulman (1992) sees the importance of this case literature, presently extremely limited (Wood & Giddes, 1999), as providing precedents, prototypes and parables which can be used by teacher educators to develop their practice. This is in contrast to the theoretical, research-based knowledge generally available to teacher educators.

Summary

This research illustrates the development of curriculum and classroom management techniques to be used in college-based courses in pedagogy which may help to solve the dilemma of practical courses given at a distance from the actual field of the classroom. It shows ways in which the particular contextual settings of these courses may be utilized to increase the relevance of courses that are often considered to be theory-laden and disconnected from the field. It suggests ways of of teaching which are inspired by constructivist theory, thus providing practical suggestions for improving the quality of

learning in college-based education courses. It also demonstrates the use of Feuerstein's theory of Mediated Learning Experience as a way to improve the focus and increase the effectiveness of learning opportunities offered in college-based education courses, as well as a tool to be used in the analysis and critique of teaching practices and teacher-student interactions. Although the subject matter discussed here is that of early childhood mathematics education, many of the problems dealt with, such as relevance, an over emphasis on theory, and the physical separation from the field of college-based courses, are relevant for classes given at a variety of levels and in different curriculum areas, such as early childhood education in general, mathematics education in general, science education, and additional subject areas as well. The contribution to teacher education, therefore, is wider than the context of early childhood mathematics education alone.

My goal in this project was to find ways to increase the educative influence of a course in early childhood mathematics education on my students' learning. The high level of student involvement that developed enabled many of the students to significantly change their perceptions and beliefs regarding mathematics, mathematics education, and their role as teachers of mathematics. These personal theories developed during the course of the year will hopefully ensure long-term commitment to an innovative approach to mathematics education. My primary concern at the beginning of the project was with the development of pedagogical content knowledge (Shulman, 1987). As the research progressed, however, the focus of my work began to change. At first I turned my attention from the ways in which I encouraged the development of the pedagogical knowledge of the individual to the ways in which the course supported the development of pedagogical norms among the participants. These norms seemed to be a crucial factor in establishing the students' openness to new ideas. Finally, however, in accordance with my goal of influencing the future practice of the individual teacher, my focus turned once again to the individual, this time looking at the ways in which the course, including the norms that developed among the students, contributed to the development and modification of individual beliefs and attitudes. The endlessly dynamic spiral of reciprocity between knowledge and beliefs, became salient to my research.

This thesis reports an action research project which was carried out during the 1997-1998 school year. In terms of subject matter introduced, the year was divided into three modules. Three major action strategies were introduced during this time, one following the other, each becoming the focus in accordance with the character of the subject

matter of that particular module. The thesis is structured in a similar way: after providing the background and the theoretical framework of my work, I present each of these modules and their accompanying modes of work. Finally, I evaluate the year as a whole, and relate it to work in the field.

CHAPTER ONE

BACKGROUND AND RATIONALE

Mathematics Education in Israel

Israel is a small country of approximately six million inhabitants. The education system here is centralized - there is one ministry of education that is responsible for the education in all the public schools in the country. Because of the small size of the country and the lack of resources, neither the public education system nor private enterprise support a wide variety of mathematics programmes and textbooks. In 1971 the Baron de Rothschild Foundation established the Institute for Educational Technology, a centre for curriculum development whose mandate was to create materials in all areas of elementary school education. Among those curriculum materials was a series of mathematics textbooks (*One, Two and Three*) which covered the whole range of the elementary mathematics curriculum. This series was developed in cooperation with the curriculum development department of the education ministry. Over a period of 25 years, this mathematics scheme gradually took over the mathematics education of almost the entire elementary school system in the country. By 1999 almost 95% of all elementary school pupils in the country were using this textbook series (Korn, 1999 - personal communication). Other than a number of very old mathematics schemes that continued to be used by a few scattered teachers, there was only one other mathematics scheme available for use in the first and second grades. Because this scheme did not include work at all the elementary grade levels, and therefore did not provide a solution to the entire school's mathematics programme, it never become widely used. It is only in the last two or three years that a number of new schemes have been developed and are beginning to be used in schools in Jerusalem and elsewhere in the country.

The approach which informs the *One, Two and Three* series, is what is called in Israel a "structured approach", (Nesher, 1982, in Hebrew). According to this approach the only manipulatives used to represent mathematical ideas are those that are built in accordance with the mathematical structure of the subject. The use of counters and/or fingers in beginning arithmetic is discouraged, as children are assumed to know how to count by the time they enter first grade, and the developers of the scheme are interested in freeing them of the need to count (Nesher, 1999). Instead, the children are provided with Cuisenaire Rods, coloured rods with no dividing lines, whose different lengths are

meant to represent the numbers from one to ten. The smallest rod, a one centimeter cube, is white, and represents the number one, with a different colour representing each of the subsequent numbers. The longest is the orange rod, which is ten centimeters long and represents the number ten. The rods are virtually the only manipulative used by the scheme in the study of all four whole number operations, with one other manipulative being added for the teaching of the base ten system. In spite of this no-counting policy, because of difficulties experienced by many children with this approach, in recent years the developers began to include stickers that can be affixed to the rods on which are stamped black dots which can be counted. These are only to be used by children who have extreme difficulty in working with the unmarked rods.

Teachers' reactions towards the scheme are varied. Although frustrated by the unclear meanings and purposes behind many of the instructions given to both teachers and children, many are willing to accept the situation in order to enjoy the convenience of having a workbook the children can use. The text, written by mathematicians who presumably know their subject and therefore what is best for children, gives them a feeling of security. If they do what is prescribed, they assume that the children will learn what is necessary for them to learn. These general classroom teachers, almost always women, are responsible for teaching mathematics as well, in spite of the fact that many of them were not particularly successful in their own careers as mathematics students. Who are they to presume that they should know better regarding their pupils' mathematics education? They therefore hand over responsibility to the writers of the mathematics scheme, and assume the caretaker role of ensuring that the children complete the pages in the workbooks at least reasonably correctly.

The approach of the developers of the scheme is one which focuses on the mathematics to be learned rather than the way in which children learn mathematics. My own approach, based on the mathematics reform literature as well as my years of experience teaching children, is one which looks at children's thinking and encourages the construction of their own mathematical understandings. This approach is in direct conflict with that of the developers of the scheme. This was the backdrop to my teaching at the college.

Recent international comparative studies carried out by the American government (the TIMSS evaluation project) showed that fourth grade and eighth grade children in Israel came well behind their counterparts in many other developed nations (TIMSS, 1995, 1999). This result would seem to corroborate my feeling that the approach towards

teaching mathematics described above does not fulfill many of the criteria that would characterize effective teaching.

The TIMSS study has been instrumental in effecting change in mathematics education in Israel. The writers of the new country-wide mathematics curriculum, Curriculum 2000, that has been in preparation over the last few years, attribute the change in its approach to mathematics education partly to the TIMSS study:

> The international tests (TIMSS) that compared [the mathematical achievement of] different countries caused the teaching of mathematics [in Israel] to become the focus of public interest. The fact that Israel is not among the countries with the highest levels of [mathematics] achievement guided the work of the writers of the new curriculum.
>
> (Introduction to Curriculum 2000)

Although research into mathematics education is well-developed in Israel, it was public outcry that gave legitimacy to both the complaints of many mathematics teachers and to the desire of researchers of mathematics education to significantly change the nature of mathematics education in the country. In hindsight it is possible to say that my own course in mathematics education may have contributed to the quickening pace of change in mathematics teaching in the country that is apparent today.

The Problem of Teacher Training

In the last thirty years a model of mathematics teacher training has developed which is based on the same constructivist principles that inform mathematics work done with children (Ball, 1990a; Simon, 1995). This model has either pre-service or in-service teachers participating in the same kinds of mathematical inquiry and problem-solving that characterize the work with children in constructivist oriented classrooms. The advantages of this kind of work with teachers are numerous - teachers have the opportunity to learn about mathematics in a way that, for many, is very different from the ways in which they were taught as children. Thus, simultaneously, they learn mathematics material in a more meaningful way than they had until now, they experience first-hand the kind of work that they themselves can put into practice in the classroom, and they can use the teacher educator as a role-model in learning to implement these kinds of classroom activity. It seemed, as such, that a successful model of teacher education had already been found, and it was with this kind of personal experience of mathematics teacher education, which had been influential enough to determine the whole course of my teaching career, that I began my work at

the college.

However, the course that I was teaching was a didactics course aimed at future early childhood teachers, teachers who would be working with children between the ages of four and eight. During my first year of teaching at the college, I became aware of the difficulty of implementing, in this context, the model of teacher training that I had experienced till then. The kinds of activities appropriate for very young children are often not suitable for use with adults. As a result, I found myself often lecturing to my students about children's ways of thinking or about activities appropriate for this age level, rather than having my students actively participate in mathematical activity. This situation caused me much concern and a strong desire to find better solutions to the problem than those I succeeded in devising at the beginning of my career as a college-level teacher.

An additional problem of teacher education caused me equal consternation. The literature is ripe with examples of ways in which various socializing influences counteract the effects that teacher education courses and programmer hope to have on their students. One of these influences is the previous experience that future teachers had as pupils themselves (Lortie, 1975; Ball, 1990a; Lacey, 1977).

> "...their experiences have often persuaded them that mathematics is a fixed body of rules, a dull and uninteresting subject best taught through memorization and drill and that they themselves are not good at math. They have developed the idea that mathematics teaching involves giving directions about what to do, assigning work, and, as one of my students wrote, 'sit at the desk and wait for people to come up for extra help or to get their papers checked'. Consequently, prospective teachers, equipped with vivid images to guide their actions, are inclined to teach just as they were taught."
>
> (Ball, 1990a, p. 12)

Another influence that research has noted is that of the schools in which novice teachers find themselves when they begin their professional teaching careers (Zeichner, 1980; Zeichner & Gore, 1990) It has been found that these teachers become quickly socialized into existing norms of practice in the schools, forsaking many of the principles and ways of acting that they had developed during their student teaching experiences.

In my own experience, I had often come across situations where students who had studied with me in the past, when beginning to work in the field either as student

teachers or as novice teachers in their own classrooms, reverted to the use of workbooks in much the same ways as the scenarios described at the beginning of the introduction. Equipped with this awareness, one of my aims for my action research was to make a systematic effort to take responsibility for my students' learning - to find ways to develop the tools and conditions which would help ensure the carryover of the work we did in the course to the students' future teaching placements. The work I had done until now had been based on a mathematics teacher education paradigm which, although offering many advantages, had proved insufficient in the context within which I was teaching. With this in mind, I incorporated three educational theories into my work: constructivist theory, which sees learning as a process in which the individual must take an active role in the creation of his or her knowledge; situated learning theory, which focuses on the place of context and social interaction in the learning process; and the theory of Mediated Learning Experience which looks at the role of the mediator in the individual's development of his or her knowledge. By aligning my work more closely with the precepts of these theories, I hoped to develop a mode of teaching which would lead to more sustainable results.

CHAPTER TWO

LITERATURE REVIEW

PRE-SERVICE MATHEMATICS EDUCATION

Inquiry into the potential influence of pre-service early childhood mathematics education on the subsequent practice of students must examine three major contributing factors: the content and context of what needs to be taught and learned, the ways in which learning takes place, and the ways in which teaching can influence present learning and future practice. The knowledge base deemed necessary for a teacher to begin teaching mathematics at any level is dependent on conceptions of education, conceptions of mathematics, views of the way learning takes place, beliefs regarding the ability of different kinds of people to learn mathematics, and more.

The Professional Standards for Teaching Mathematics (NCTM, 1991) arose from the reform view of mathematics teaching as expressed by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards published in 1989 (NCTM, 1989).

> Central to the Curriculum and Evaluation Standards is the development of mathematical power for all students. Mathematical power includes the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity and inventiveness also affect the realization of mathematical power.
>
> (NCTM, 1991, p. 1)

This view of mathematics and mathematics education, to which I, too, subscribe, fostered six standards for the professional development of teachers (NCTM, 1991, p. 123):

1. *Experiencing Good Mathematics Teaching.* As discussed above, in Israel as well as in many parts of the world, the mathematics experienced by most future teachers during their career as schoolchildren is largely at odds with the spirit of this new view of mathematics and mathematics education. One of the functions of the didactics class, therefore, is to allow students to experience a different kind of mathematics teaching,

one where the learners are actively involved in meaningful mathematical thinking and activity, and in which all students can participate and succeed. It would seem reasonable to expect that a teacher could not teach a new kind of mathematics if s/he had not first experienced that kind of learning first-hand.

> Our view of mathematical learning influences how we think about teaching. If we believe that education is mainly learning facts and procedures quickly and efficiently, if we believe that only certain students need or can learn mathematics, or if we believe that people are born with the ability to do or not to do mathematics, then our view will conflict with the development of understanding.
>
> (Lindquist, M.M. 1997, p. xiv)

Part of the mandate of this didactics course, therefore, was to allow my students to experience alternative ways of learning mathematics.

2. *Knowing Mathematics and School Mathematics.* The lack of knowledge of mathematics content would seem to be one of the major reasons that teachers are often so willing to turn over responsibility for their mathematics curriculum to textbook writers who have no knowledge of their students or the contexts within which they teach and learn. The degree to which teachers feel comfortable with and confident about teaching mathematics affects both what and how they teach. Compounded with this, and somewhat contradictory to it, is the fact that early childhood teachers are often not aware that there is mathematical knowledge or knowledge of school mathematics which could be useful to their teaching. Students often begin didactics courses believing that all they need are a few teaching tips to help them teach what they already know.

> ...the importance of seeing themselves as subject-matter experts is not emphasized to teachers- especially teachers in the early and middle grades: they fall into believing the old saw that "those who can, do. Those who can't, teach." Teachers are not encouraged to seek the knowledge and understanding that would allow them to teach academically rigorous curricula.
>
> (Bransford, Brown & Cocking, 1999, p. 8)

In order for a teacher to impart subject matter content in a way that will be meaningful for students, the content must be meaningful to the teacher. The question arises, particularly in the case of early childhood teaching, as to what content knowledge is necessary for the teacher in order to achieve this goal. Leinhardt et al. (1991) make it clear that one does not have to be a mathematician in order to teach school-level curriculum.

> We do not mean the knowledge of advanced topics that a mathematician might have. The point here is that a teacher will not become a better teacher *simply* by taking increasingly more advanced math courses in topics such as Chaos (although this might go a long way toward increasing love of mathematics), but will become better if the depth of knowledge about a particular school topic...is enriched. This deep knowledge includes knowledge about ways of representing and presenting content in order to foster student learning or construction of meaningful understanding.
>
> (Leinhardt et al., 1991, p. 88)

Shulman stresses the importance of understanding the structure of the content. He cites Schwab's (1978) construct which includes both substantive and syntactic structures.

> The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established...
>
> Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to their propositions , both within the discipline and without, both in theory and in practice
>
> (Shulman, 1991, p. 9)

Although the course I was teaching was a course in mathematics pedagogy, the mathematical knowledge of this population of student teachers is often insufficient. It was incumbent on me, therefore, to address mathematical content knowledge as well. Because the course dealt with early childhood, however, the focus was on providing a deeper understanding of those mathematics topics more immediately relevant to the learning of small children. To the extent that this subject matter could be taught in novel ways that were in keeping with the spirit of the new view of school mathematics, the impression that their study left on my students could be a first step in their understanding the novelty of this approach. This could begin to raise their awareness of the many ways in which their prior understandings of both mathematics as a discipline and school mathematics needed to critically evaluated, modified and augmented.

3. *Knowing Students as Learners of Mathematics.* The NCTM document holds that in order to teach mathematics to young children it is imperative to know how they think, what their interests are, and what prior experiences they have had of mathematical situations.

The study of general principles of teaching and learning is insufficient for

> teachers of mathematics because it does not include consideration of the nature of mathematics and of current research on children's mathematical thinking and its implications for instruction.
>
> (NCTM, 1991, p. 144)

Traditional mathematics teaching most often views mathematics as a body of external knowledge to be passed on to the learner. Constructivist theory, on which the NCTM recommendations are based, challenges that view of mathematical knowledge by holding that any reality can only be known through the eye of the learner. Learning is and must always be based on our own prior experiences and understandings.

> Constructivism derives from a philosophical position that we as human beings have no access to an objective reality, that is a reality independent of our way of knowing it. Rather, we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge.
>
> (Simon, 1995, p. 115)

Teachers can begin to understand the importance of seeing mathematics learning as the reinvention of mathematics by each individual (Kamii, 1985) by becoming aware of the richness and diversity of children's mathematical thinking. Until then they will continue to teach mathematics by giving the children techniques and algorithms which allow them to solve problems without thinking or understanding.

4. Knowing Mathematical Pedagogy. This NCTM standard points to teachers' knowledge of and ability to use and evaluate instructional materials and resources, ways of representing mathematical concepts and procedures, instructional strategies including classroom discourse, and ways of evaluating children's mathematical understanding. In my eyes, this fourth standard would seem to include but go beyond the first three. It is only through having experienced good mathematics teaching, understanding the nature of mathematics and school mathematics, and understanding their students as learners that a teacher can develop an understanding of mathematical pedagogy. Shulman (1986a, 1986b) calls this knowledge "pedagogical content knowledge", knowledge which

> ...goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching*. I still speak of content knowledge here, but of the particular form of content knowledge that embodies the aspects of content most germane to its teachability...
>
> Pedagogical content knowledge also includes the understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with

> them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners because those learners are unlikely to appear before them as blank slates.
>
> (Shulman, 1986, pp. 9-10)

Leinhardt (1991) discusses these same points, but calls them subject-matter knowledge:

> This deep knowledge...also includes knowledge of what the students bring to the learning situation, knowledge that might be either facilitative or dysfunctional for the particular learning task at hand. This knowledge of students includes their strategies, prior conceptions (both "naive" and instructionally produced), misconceptions that students are likely to have about a particular domain, and potential misapplications of prior knowledge.
>
> (Leinhardt, 1991, p. 88)

This difference in terminology emphasizes the extent to which Shulman's construct of pedagogical content knowledge incorporates all of the first four professional standards cited to date. It, along with the continuing professional development of my students as teachers as depicted in the two final NCTM standards, were the two main foci taken into account during the course of my research.

The final two standards have to do with the role of teacher education in the development of teachers' pedagogical content knowledge and the role of teachers' own continuing efforts to learn and develop.

5. *Developing as a Teacher of Mathematics.* This standard refers to the opportunities that need to be provided for pre-service and in-service teachers to observe, analyze and evaluate their own assumptions about the nature of mathematics, a range of approaches to mathematics teaching, and the appropriateness and effectiveness of their own teaching. It also refers to the opportunity to work with diverse individuals or groups of students with guidance from mathematics education professionals.

> This standard addresses issues that are at the heart of teaching. The goal of teacher education is to "light the path" for those who follow, providing directions on how to plan and teach mathematics. It is the practice of teaching, the growing sense of self as teacher, and the continual inquisitiveness about new and better ways to teach and learn that serve teachers in their quest to understand and change the practice of teaching.
>
> (NCTM, 1991, p. 160)

Civil (1993) shows that for students taking a course similar in conception to my own,

both the students' view of mathematics and their concern about teaching mathematics in an actual classroom reality perceived by them as being at odds with what was taught in the course, made it difficult for them to change their views of mathematics education. I envisioned three major ways to facilitate change in their views. The problem as presented by Civil was the major focus of my work, which I addressed through the use of mediated constructivist-inspired work on the pedagogy of mathematics and attempts at situating my students' learning to the greatest extent possible.

6. *The Teacher's Role in Professional Development.* This standard refers to the responsibility that teachers accept for their own professional development. It refers to the continuing examination of their own present practice and possible alternative approaches through experimentation, reflection, discussion with colleagues, and reading the professional literature. My own course could contribute to this future development by encouraging the critical appraisal of both their own work and that of others working in the field of mathematics education, and by enabling them to experience the satisfaction and excitement that comes with struggling with new educational ideas and recognizing the place these ideas can have in their own educational framework.

BECOMING A PROFESSIONAL

Much research has been done on the professional practice of experts in a variety of domains - from that of master chess players to that of expert athletes and musicians. Among the many reasons to look into expert practice, its relevance for this research project lies in both the idealized pictures as well as the realistic models of effective teaching that it can provide.

> The performances of experts, though not necessarily perfect, provide a place to start from when we instruct novices. The experts' performance provides us, as Glaser (in press, a) has noted elsewhere, with a temporary pedagogical theory , a temporary scaffolding from which novices may learn to be more expert.
>
> Another reason to study experts is that they sometimes provide exemplary performances from which we can learn.
>
> (Berliner, 1986, p. 6)

I have chosen to look at those characteristics of expert practice that are relatively easy to observe, discussing the possible reasons for the development of these features, and then to show how these understandings may be helpful in suggesting improved practice in pre-service teacher education.

From the literature which has looked at expert-novice differences in teaching as well as in other domains it is possible to classify expert teacher practice into three major categories:

*the way in which experts perceive and interpret classroom events and data
*the automatic character of much of expert teaching
*the relative flexibility of their teaching which, nonetheless, is constrained by relevant information of students, subject matter, and environment

Expert and Novice Information Gathering

The classroom is a setting containing a plethora of information about students, curriculum, and context that must be taken into account by the teacher. It follows that the ability to function as a teacher in this complex cognitive domain (Leinhardt & Greeno, 1986) is a significant achievement. Berliner indicates, while possibly underestimating this complexity:

> We have decided from observation and reflection that two large domains of knowledge must be readily accessed to be an expert pedagogue. We have stipulated these two domains of knowledge to be subject matter knowledge and knowledge of organization and management of classrooms...at least [these] two complex and extensive knowledge domains must be integrated at all times...
>
> (Berliner, 1986, p. 9)

The literature shows that expert teachers, as opposed to novice, more efficiently perceive and interpret information about their students (Calderhead, 1986), about the classroom environment (Carter et al, 1988; Berliner, 1994), and about unexpected occurrences (Berliner, 1994, Tan, S., 1997). Carter et al, (1988) found experts to be more selective in their attention to classroom events, and to focus their reflections on student understanding and on their instructional goals. They seem to have a better sense of what was typical, and therefore are able to ignore information less relevant to the educational goals of the lesson.

Peterson and Comeaux (1987), in a study which compared novice and expert high school teachers in their recall and analyses of problematic situations in their teaching, found that experienced teachers more often discussed problems in terms of principles

and procedural knowledge of the classroom, gave more elaborate answers to interviewers' questions, and gave more justifications for instructional decisions or comments that they made regarding their teaching. Experts were also found to more frequently make hypotheses based on available evidence (Berliner, 1986). Because experts have a rich repertoire of instructional activities, they are more efficient in their lesson planning. They plan in detail where needed, but not when they have readily available curriculum scripts or explanations (Leinhardt et al., 1991).

Explanations for Differences

There have been a number of explanations put forward for these observed differences between expert and novice teachers. One of these is that expert teachers have better-developed knowledge structures relating to classroom events. Teaching, a complex cognitive skill (Leinhardt & Greeno, 1986), may gradually be developed and refined through increased experience. This development is expressed in the form of schemata that are built up by finding patterns and seeing relationships between events and concepts that are met in the course of teaching. In this way knowledge of teaching is formed into chunks of information which form complex hierarchic structures (Schempp et al., 1998). These structures may be easily perceived and accessed, thus freeing the individual to perceive and take into account increased amounts of information, including occurrences that may previously have gone unnoticed.

Berliner (1994) focuses his discussion of experts' ability to perceive and recognize information in classroom teaching in two directions. First he looks at the store of patterns that experts have built up through long and rich experience of interactive teaching. This pre-existing organization of information facilitates the present perception of patterns, allowing the teacher to connect present information with a readily available range of prior understandings.

> It is...said that experts have extraordinarily fast and accurate pattern recognition capabilities. These recognition skills appear to act like schema instantiations. The recognition of patterns reduces the cognitive processing load for a person. Sense is instantaneously made...
>
> (Berliner, 1986, p. 11)

From Berliner's description of this ability it would seem as though experience may be a prerequisite for this kind of perception:

> We regard the reading of a classroom, like the reading of a chessboard, to be in part a pattern recognition phenomenon based on hundreds and thousands

> of hours of experience.
>
> (ibid.)

Doyle (1977), however, in a study comparing the teaching of more and less successful student teachers, shows that there are student teachers who do, to some extent at least, display this ability.

> ...successful student teachers tended to classify individuals in terms of their potential for disruption, skills in classroom tasks, inclinations to participate in lesson activities, etc. They seemed to know that the movement of some students around the room to secure supplies or sharpen pencils could be ignored whereas the movement of other students required careful monitoring.
>
> (Doyle, 1977, p. 54)

Doyle, although pointing to this variability in the work of student teachers, does not suggest ways of encouraging the development of this kind of perception that may be useful for teacher educators. One of the purposes of my research was to investigate what these ways might be.

Berliner's second focus is on the greater likelihood that experts have of connecting present phenomena with pedagogical concepts. He shows how expert teachers are more cognizant of information which has instructional significance, and attributes this to the concepts and principles they use to impose meaning on classroom phenomena. Doyle again sees this ability as existing in certain student teachers as well, and explains it by referring to their chunking and differentiation abilities. Chunking refers to the classification of information into categories that can then be accessed as single chunks of information rather than as individual pieces. It reduces the load on the short-term memory capacity of the teacher, freeing her or him to attend to additional information. Differentiation refers to the capacity to grasp the relative significance of pieces of information.

> In sum, successful student teachers transformed the complexity of the environment into a conceptual system that enabled them to interpret discrete events and anticipate the direction and flow of classroom activity.
>
> (ibid.)

Yinger (1987) refers to the development of the ability to perceive and interpret information as the development of a language of practice. He points out the importance of viewing professions as cultures with their own shared "perceptions, conceptions, and acceptable actions" (p. 295).

> ...a language of practice is a set of integrated patterns of thought and action. These patterns themselves constitute a kind of syntax and semantics for action. The words and phrases in this language are behavior, activities and routines. As such a language of practice is usually found in a practitioner's action rather than in one's speech. It is not heard, but seen and felt.
>
> (Yinger, 1987, p. 295)

Livingston and Borko (1989) characterize this complex knowledge of teachers as pedagogical reasoning and pedagogical content knowledge, in which subject matter knowledge is integrated with an understanding of teaching and thus transformed into "forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p.15). As mentioned earlier, pedagogical content knowledge incorporates subject matter knowledge with knowledge of children and classrooms.

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations - in a word, the ways of representing and formulating the subject that makes it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some which derive from research whereas others originate in the wisdom of practice.
>
> (Shulman, 1986a, p. 9)

Automaticity

There would seem to be at least two different ways in which expert practice appears to be automatic. The first is in connection with routine procedures that repeat themselves many times and result in seemingly effortless, unthinking performance. Leinhardt & Greeno (1986) compared the opening homework review of expert and novice mathematics teachers and showed how the expert, as opposed to the novice, was able, in less time, to get all the homework corrected while checking attendance, ascertaining which students had completed the homework, and determining which students were in need of help. Experts seemed to have routines that were so well-practised that they gave the impression of working out of habit. Hawkins & Sharpe (1992) analyzed and compared videotapes of expert and novice physical education teachers and found that experts more frequently used repeating chains of behaviour. Berliner (1994) reports a study in which novice, advanced beginner and expert teachers were asked to teach a lesson on probability to a group of students that they were not familiar with. The expert

teachers were not happy about taking part in the task partly because they did not have the chance to establish routines of behaviour with this new group. It seems that they were aware of the importance of routine in the teaching of smooth-flowing lessons.

> The well-practised routines of expert telegraphers, surgeons, ice-skaters, tennis players, and concert pianists (Bloom, 1986), no less than expert teachers, are what give the appearance of fluidity and effortlessness to the performance of experts. What looks to be so easy and seems so clumsy for the novice is the result of thousands of hours of reflective practice, experience from which learning derives.
>
> (Berliner, 1994, p. 14)

The second way in which experts' practice seems to be automatic is in the fluidity of their response to unfolding events in their classroom teaching (Schempp et al, 1998; Berliner, 1986; Eraut, 1994). Dreyfus & Dreyfus' (1986) description of expertise provides an additional description of automaticity:

> For the expert, not only situational understandings spring to mind, but also associated appropriate actions. The expert performer, except of course during moments of breakdown, understands, acts, and learns from results without any conscious awareness of the process. What transparently *must* be done *is* done.
>
> (Dreyfus & Dreyfus, 1986, p. 324)

Relating this fluidity of practice to the ability of experts to perceive and interpret information, it would seem that the complex, hierarchically structured knowledge of experts allows them to access and utilize information so efficiently that their actions are often perceived as purely intuitive and automatic. Eraut (1994) considers this kind of action to be semi-automatic:

> Managing a class of children involves a myriad of rapid decisions made on the spur of the moment in response to rapid readings of the situation and the overall purpose of the action. I call the latter semi-automatic because all the decision-making is very rapid; there is no time for deliberation during the action itself.
>
> (Eraut, 1994, p. 239)

Flexibility

There are two major ways in which professionals may be seen as being flexible in their practice. The first relates to experts' case knowledge, developed through their experience with educational situations, which allows them to better understand and more easily respond to current events. The second relates to their repertoire of

instructional strategies which may be called into use when appropriate. These allow them to anticipate different scenarios when they are in the process of planning.

Case Knowledge

Livingston & Borko (1989) have shown a relationship between teachers' schemata, built up through extensive case knowledge, and their ability to improvise in the course of their interactive teaching. Teaching is seen as a complex cognitive skill through which the teacher forms schema which subsequently form the base for future perception, understanding and problem-solving. These knowledge structures are complex and hierarchically structured, allowing expert teachers to more easily interpret classroom events and take them into account in their on-the-spot decision making.

Carter et al. (1998) , in a task in which they showed expert and novice teachers a series of slides of interactional teaching, found that because of their extensive experience, experts were able to distinguish automatically between important relevant information and information that is more common-place and therefore trivial in the circumstances.

> ...data suggest that experts use their experience and domain-specific knowledge of pedagogy to envelop with meaning certain of the events they observe in the classroom environment as well as to find many events so ordinary or typical that they literally find them meaningless.
>
> (Carter et al., 1988, p. 28)

Once situations are seen as "typical" no further energy or time need be invested in their consideration, allowing experts to be more attuned to the relevant requirements of the situation, and to orchestrate their performance in accordance with them.

Similarly, Hawkins and Sharpe (1992), using field system analysis, found that although experts had more automatic, predictable routines, their teaching on the whole was more flexible (made up of more distinct behavioural elements produced in many different orders).

Much of novice teaching is rule-bound. Shulman (1986a, 1986b) suggests that teacher knowledge comes in three forms: propositional knowledge, case knowledge and strategic knowledge, and characterizes the main form of knowledge taught to students as propositional. Empirical propositions are generalizations that derive from empirical findings: Children are more interested when active. Moral propositions derive from value positions: All children must be viewed as capable of learning. As a whole,

propositions are unilateral - they have "the deficiency of turning the reader or user toward a single, particular rule or practical way of seeing" (Shulman, 1986a, p. 12).

For practising professionals, their personal case knowledge is acquired through years of experience which, when reflected on, has the potential to result in the strategic understanding prerequisite to the flexible practice characteristic of the expert.

> It appears that experts, perhaps by the very nature of acquiring expertise through extensive and varied teaching experiences, have a rich store of classroom knowledge about both students and events, and they use that knowledge to understand and explain classroom phenomena.
>
> (Carter at al., 1988, p. 30)

Shulman (1986a) refers to the use of cases in teacher education as a way of arriving at strategic understanding.

> When strategic understanding is brought to bear in the examination of rules and cases, professional judgment, the hallmark of any learned profession, is called into play. What distinguishes mere craft from profession is the indeterminacy of rules when applied to particular cases.
>
> (Shulman, 1986, p. 13)

It is this professional judgment that may be lacking not only in "mere craft", but also in the thinking of novice teachers - according to Dreyfus & Dreyfus (1986), for the beginner, doing a good job means following learned rules as closely as possible. The remedy for the unilaterality of propositional knowledge can be found in strategic understanding which takes into account both propositional knowledge and case knowledge.

Repertoire of Instructional Strategies

This case knowledge can lead to the second source of flexibility, the knowledge of a wide range of instructional strategies that may be called upon both in planning and in on-the-spot teaching. Shulman (1986a) refers to this as curricular knowledge, and holds that teacher educators are even more delinquent with respect to this kind of knowledge than they are in respect to pedagogical knowledge. The existence of an extensive instructional repertoire benefits interactive teaching in that it allows experts to choose, in the heat of the moment, from a large selection of appropriate courses of action. Livingston & Borko suggest the difficulty that novices must experience when faced with the challenge of developing the knowledge structures necessary for effective

themselves experienced as children, or that they have much to learn before they can become competent teachers. Novices, tending to accept too much as given, are often not in the position to begin to reflect on their own teaching.

Dewey's second subprocess of reflection is

> an act of search or investigation directed toward bringing to light further facts which serve to corroborate or to nullify... belief.
>
> (Dewey, 1997 (1910), p. 9)

Having come upon a perplexing situation, the reflecting individual considers different reasons for deciding whether a belief is correct or incorrect. This is the process of reflection. It would seem that in a professional situation the thinker must have enough previous information to both recognize the problematic nature of the situation and to relate it to existing beliefs. Are those conditions of reflective thought existent in novices? The research on novice-expert differences would seem to indicate that it is not.

Calderhead (1981) found that experts had the conceptual structures necessary to make sense of classroom situations that beginners lacked. The meaning that they were able to extract from situations was of a different kind and at a different level from that of novices. The work cited above on the complex knowledge structures of experts which allow them to perceive and interpret information more efficiently, is relevant here. Peterson & Comeaux's findings (1987), mentioned above, support the hypothesis that experienced teachers have better developed schemata relating to classroom events and show that these schemata affect their ability both to recall events and to analyze problem-solving situations. Berliner (1986a, 1986b) reports on research which shows that experts categorize problems at a higher level than do novices, looking past the surface characteristics that might attract the attention of novices.

Experts not only reflect more effectively, but have more of an inclination to reflect on and infer meaning from classroom events in the first place. In their research, Carter et al. (1988) show how experts tend to make more hypotheses based on available visual evidence. Similarly , Berliner (1986) reports work done by his students which shows that novices make fewer inferences from information than do experts.

Schempp et al. (1998) compared the reflective practice of novice teachers to that of competent teachers rather than experts. They found that in planning lessons, competent teachers were aware of variation in pupils' understandings, based their plans on informal, subjective assessments of this understanding and continued appraising pupils

planning:

> Experts' planning can be described as a process of combining information from existing schemata to fit the particulars of a given lesson. Because experts have well-developed and easily accessible schemata for aspects of teaching such as instructional activities, content and students, they are able to plan quickly and efficiently. Novices, on the other hand, often have to develop, or at least modify and elaborate, their schemata during planning. Their schemata for pedagogical content knowledge seem particularly limited.
>
> (Livingston & Borko, 1989, p. 39)

Their repertoire knowledge allows expert teachers , in planning their lessons, to better anticipate classroom scenarios and create realistic contingency plans (Livingston & Borko, 1989; Peterson & Comeaux 1987; Clark & Peterson, 1986). The result is greater flexibility in interactive teaching.

> Presumably, schemata for classroom events and life in a...classroom affect the teacher's perception of events during interactive teaching, affect the teacher's perception of students, enhance the teacher's understanding of events that may occur during interactive teaching, and aid the teacher in problem solving and decision making during interactive teaching.
>
> (Peterson & Comeaux, 1987, p. 329)

The study of instructional strategies which would begin to form my students' repertoire was a well-developed aspect of my teaching prior to my research and had received much positive feedback from my students in the past. The use of cases, which was only in its infancy in my previous teaching, began to develop during my research. This was influenced both by the literature cited above and by my increasing awareness of the importance of situating my students' learning, to be discussed later.

Reflection

Dewey defines reflective thought as follows:

> Active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends...
>
> (Dewey, 1997 (1910), p. 6)

According to Dewey, one of the subprocesses that is involved in every reflective operation is "a state of perplexity, hesitation, doubt". Unless this element is in play, there is no reflective thought. Ball (1990a) has shown that student teachers are often unaware of the fact that there may be a different kind of teaching than that they

understanding throughout the unit. Novices, on the other hand, neither perceived nor analyzed what pupils know, and assumed that they had no prior knowledge of the subject to be taught.

A major challenge of my research was to promote a frame of mind in my students that would support a critical reflective attitude toward their own work and toward received pedagogical suggestions and curricula. The major ways in which this was attempted were based on Feuerstein's idea of the mediation of meaning, to be discussed below, and through my attempts to have my students reflect on the lessons they gave in the classroom.

Eraut (1994) sets out a number of assumptions on which the concept of the teacher as a reflective practitioner is based. These assumptions summarize well the findings of the characteristics of professional practice presented above.

> *A teacher needs to have a repertoire of methods for teaching and promoting learning.
> *Both selection from this repertoire and adaptations of methods within that repertoire are necessary to best provide for particular pupils in particular circumstances.
> *Both the repertoire and this decision-making process within it are learned from experience.
> *Teachers continue to learn by reflecting on their experience and assessing the effects of their behaviour and their decisions.
> *Both intuitive information gathering and routinized action can be brought under control through this reflective process and modified accordingly.
> *Planning and pre-instructional decision making is largely deliberative in nature. There is too little certainty for it be a wholly logical process.
> *These processes are improved when small groups of teachers observe and discuss one another's work.
>
> (Eraut, 1994, p. 231-232)

Competence in Student Teachers

The challenge of my action research was to see if I could find ways to assist student teachers in their second year of teacher training, to become competent as teachers of early childhood mathematics. A review of some of the points raised regarding expertise in the previous section can point to a number of areas where well-devised teacher education may have the potential to reach this goal, as well as other areas which would seem to require a period of actual experience before they can be developed.
Dreyfus & Dreyfus (1986) have proposed a series of five stages by which practitioners may gradually develop from novices to experts: novice, advanced beginner, competent,

proficient and expert. I have already described many of the characteristics of the final, expert stage, and will now look at the first three of these stages to help in determining ways of working with student teachers which may support their progression from the novice stage to either the advanced novice or the competent stage.

The Novice Stage

Before describing the novice stage, it is important to note that, in looking at this stage of professional development, the population described by Dreyfus & Dreyfus are beginning practitioners who have studied in traditional training programmes. In these programmes they were first taught theoretical, context-free knowledge about the profession, and only subsequently began practising this theory.

> Normally, the instruction process begins with the instructor decomposing the task environment into context-free features which the beginner can recognize without benefit of experience. The beginner is then given rules for determining actions on the basis of these features, like a computer following a program.
>
> (Dreyfus & Dreyfus, 1986, p. 321)

They hold that because these beginning practitioners lack on over-all view of the practical situation, at this initial stage they are most concerned with the extent to which they are successful at following the given rules, rather than looking at the practical results of their activity. The energy expended in following more than a few of these rules will be so great that novices will be severely limited in their ability to talk about what they have done or listen to advice. Berliner agrees with and elaborates this stage in regard to teaching:

> Understanding of the commonplaces and some context-free rules are what is needed to begin to teach. The behavior of the novice, whether automobile driver, chessplayer, or teacher is usually rational, relatively inflexible, and tends to conform to whatever rules and procedures they were told to follow.
>
> (Berliner, 1994, p. 5)

The Advanced Beginner Stage

At this stage, as a result of increased experience, the practitioner begins to take note of, or is pointed out and eventually recognizes, additional aspects of the situation. According to Dreyfus & Dreyfus, it is now possible for the instructor to refer to situational aspects in addition to the context-free features recognizable by the novice.

> The advanced beginner confronts his environment, seeks out features and

> aspects, and determines his actions by applying rules. He shares the novice's minimal concern with quality of performance, instead focusing on quality of rule following. The advanced beginner's performance, while improved, remains slow, uncoordinated, and laborious.
>
> (Dreyfus & Dreyfus, 1986, p. 322)

In regard to teachers, Berliner (1994) suggests that there is less of a difference between the practice of novices and advanced beginners than hypothesized. One possible explanation for this may be the familiarity of future teachers with the life of the classroom - although teachers' childhood experiences might have been very different from the kind of teaching envisioned by teacher educators, because of their familiarity with situational aspects of the classroom, teachers may almost never be complete novices. Nonetheless it is questionable whether this prior experience can actually contribute to student teachers' development, or hamper it instead (Ball, 1990).

The Competent Stage

According to Dreyfus & Dreyfus, the competent level is achieved at the point at which so many features and aspects of the professional situation have been collected that the practitioner has no choice but to find a way to organize them in a hierarchical system. It is then that the practitioner must learn to

> choose a plan, goal or perspective which organizes the situation and ...examine only the small set of features and aspects that he has learned are the most important given that plan...
>
> (Dreyfus, 1986, p. 322).

They hold that practitioners at this level, as opposed to novice and advanced beginner, begin to have a feeling of responsibility for their practice.

> The competent performer..., after wrestling with the question of a choice of perspective or goal, feels responsible for, and thus emotionally involved in, the result of his choice. An outcome that is clearly successful is deeply satisfying and leaves a vivid memory of the situation as seen from the goal or perspective finally chosen.
>
> (ibid., pp. 322-323)

From this standpoint the teacher can see the situation as a whole and perceive certain aspects as relevant and important while allowing others to move to the background. Berliner writes:

> This is the stage in which teachers learn not to make timing and targeting

> errors, because one has learned through experience what to attend to and what to ignore in the classroom.
>
> (Berliner, 1986, p. 7)

Schempp et al. (1998) have specifically compared the practice of teachers who are seen to be at the novice and competent levels. They found major differences between the two levels in three areas: their perceptions of student learning difficulties, their conceptions of the knowledge necessary for teaching and the knowledge they want their students to know, and their ability to reflect on their practice. Regarding the first of these, novices most often attributed student learning difficulties to the learners' personal characteristics or social backgrounds, while competent teachers attributed them more to the structure and organization of the lessons they experienced. This finding is in keeping with Dreyfus & Dreyfus' characterization of competent professionals' tendency to accept responsibility for their practice, and the control they are beginning to feel as a result of their making their own choices regarding their goals and plans for their lessons.

Regarding the second area of difference, the adage, "The more you know, the more you know that you don't know" comes to mind. Competent teachers were more cognizant of their lack of knowledge in subject matter content and consequently felt a greater need to learn.

> Perhaps reflecting a greater confidence in themselves as teachers, but less secure in their knowledge than novices, competent teachers had no qualms about using knowledgeable students for class demonstrations and explanations.
>
> (Schempp et al., 1998, p. 16)

Novices often neutralize their lack of content knowledge by refraining from asking open questions and by using activities which can be successful in spite of their lack of content knowledge. According to Schempp et al., competent teachers were more concerned about becoming knowledgeable about the subject themselves.

There were also differences in the way novices and competent teachers justified subject matter they taught to their pupils. While novice teachers called on tradition and authority as reasons to justify the teaching of a lesson, competent teachers more often appealed to logical or technical considerations.

Although they refer to it as reflective practice, Schempp's last difference noted between competent and novice teachers specifically refers to their understanding of the importance of assessing their students' knowledge in a particular subject domain in order to teach them, while novice teachers perceived little variation in the knowledge of

their pupils.

> Competent teachers used ongoing appraisals of student learning to identify learning difficulties and design supplemental, remedial, or enrichment activities. Furthermore, through student performance observations, competent teachers constantly used subjective evaluation in making decisions regarding subsequent teaching activities.
>
> (Schempp et al, 1998, p. 17)

Implications for Teacher Education

Some suggestions can be gleaned from the novice-expert literature for teacher education. Peterson & Comeaux (1987) hypothesize that, if there is sufficient knowledge about expert teachers' schemata, teacher educators, rather than leaving it up to experience, might be able to facilitate the development of more sophisticated schemata on the part of student teachers during their pre-service training. They point out, however, that simply knowing about the processes involved in expert teaching does not mean that one is able to put it into practice oneself. Their suggestion is that it would be necessary to break down the explanation in such a way "that the novice can assimilate and retain the information" and to give him or her "many opportunities to practice the procedure" (p. 330). I would hold that this description of teacher education relates to that practice which does not yet put into practice either the ramifications of constructivist theory regarding the way human beings build their own knowledge (von Glasersfeld, 1990; Confrey, 1990; Ernest, 1991), nor research findings which indicate the importance of mediation in ensuring the occurrence of meaningful learning in the individual (Feuerstein, 1980, 1997).

The two foci that Berliner (1994) uses in analyzing expert teaching can be helpful here once again. First, experts' existing schemata make it easy for them to recognize patterns and make connections between present events and past, thus enabling them to continue to construct their professional understanding. It would seem that experience and/or extensive study of cases is necessary for the perception of patterns - one needs a store of specific instances in order to see them as forming a pattern. Berliner's second focus is on the connecting of classroom phenomena with pedagogical concepts. Student teachers, if left on their own to make sense of field experiences, may not have the knowledge structures necessary to learn effectively from their experiences.

Feuerstein's work on mediated learning (1980, 1997) is helpful in understanding the ramifications of these for pre-service teacher education. According to this theory,

human beings are not always able to learn directly from their experience. Through mediation on the part of a more knowledgeable individual, learners can be helped to both perceive and learn from their surroundings in ways that they could not do on their own. For novice teachers, whose schemata relating to the teaching enterprise are not yet well-developed, the recognition of patterns and the connection of classroom phenomena with pedagogical concepts may not occur without this mediation.

Berliner recommends the use of simulated field experiences, in either written or videotaped form, in order to form the connections between real-life classroom phenomena and abstract pedagogical principles. These cases can also provide the exposure necessary to begin to discern patterns in classroom phenomena. Livingston & Borko suggest that cases might be even more appropriate at the initial stages of teacher education than direct observation or participation in classroom work in that they provide a shared experience that can be analyzed and reflected on with the support of the college teacher's more developed educational schema. The teacher's mediation of the meaning and importance of the observed events can serve to focus students' attention on discernible patterns and to connect theory with practice.

It has also been found that not all experienced teachers reach the level of expert or even proficient teaching (Berliner, 1986). If a large amount of experience is not in itself necessarily sufficient, then there must be ways to take advantage of available experience which render it more significant in the formation of understanding. As such, it may be possible, through the use of Mediated Learning Experience (Feuerstein & Feuerstein, 1991) and through increasing students' ability to reflect, to increase the salience of the limited experience of student teachers just as it would seem necessary to do for some experienced teachers as well.

Another implication of pre-service teacher education may be discerned from Livingston & Borko's view of the importance of structuring and sequencing student teaching experience so that

> ...the task demands are appropriate to the novices' level of readiness. Further, these activities must be designed explicitly to help novices develop and elaborate knowledge structures for teaching and pedagogical reasoning skills. A key to designing such experiences may be the maintenance of a balance between support and challenge (Bolin, 1988; Calderhead, 1988).
>
> (Livingston & Borko, 1989, p. 39)

Regulating task demands brings to mind apprenticeship situations studied in the

literature on situated learning (Lave, 1988) in which apprentices gradually take on responsibilities which are in keeping with their current abilities. This recommendation is in keeping with Collins et al's (1989) concept of cognitive apprenticeship which, rather than focusing on physical skills and processes, is concerned with the development of cognitive and metacognitive skills and processes. These will be discussed further later on in this chapter.

Although Livingston & Borko's recommendations relate to full-time field experiences, not for college-based courses, there is at least one suggestion that can be modified for college classroom use.

> In an ideal situation, the cooperating teacher and university supervisor fulfill complementary roles...Working cooperatively and using shared language as an indicator of shared concepts, they help student teachers to develop and integrate schemata by examining the immediate situation in terms of craft knowledge as well as theoretical and empirical knowledge.
>
> (Livingston & Borko, 1989, p. 41)

The potential exists in college-based teacher education courses to combine the theory and practice in the same venue: as mentioned above, because teacher educators are themselves involved in practising the same profession that is being taught to students, their own practice can provide both a role model and real cases for the study and analysis of teaching practice. If teacher educators, advocating effective teaching practices to be used with children, take advantage of these practices in their own teaching, they can minimize the theory/practice divide, involve students in real teaching/learning situations, and reflect on these along with the students, thus taking advantage of the teacher educator's complex knowledge structures. They thus can both model and involve the students in pedagogical thinking .

These references to situated learning theory and to Mediated Learning theory both support a constructivist view of learning as an activity carried out by the learner in accordance with his or her own interests and previous understandings. Together these three theories, to be discussed in the following sections, form the basis on which it is possible to conceptualize the kind of teacher education that may better support the development of student teachers in their journey to becoming competent teachers of mathematics for young children.

CONSTRUCTIVISM AND SITUATED LEARNING

Constructivism

The general approach which has informed my work both as a teacher and as a teacher educator is that of constructivism. Constructivism, in its most basic, and, among practising teachers, possibly its most widespread form, sees learning as something that must be carried out by the learner him or herself, and that is rooted in and grows on the previous knowledge and conceptions of the individual.

> Constructivist learning theory has two basic premises: (1) learning takes as its starting point the knowledge, attitudes, and interests students bring to the learning situation, and (2) learning results from the interactions between these characteristics and experience in such a way that learners *construct* their own understanding , from the inside, as it were.
>
> (Howe & Berv, 2000, p. 30-31)

Constructivist theory has guided and/or given meaning to much of the practice of mathematics education internationally during the last thirty years. It supports the use of manipulatives and experiential activities which provide the base on which the learner can develop his or her own mathematical understandings. Skemp (1971), in his discussion of the formation of mathematical concepts, distinguishes between primary concepts, those which are based on sensory and motor experience of the external world, and secondary concepts, those which are abstracted from other concepts.

> In general, *concepts of a higher order than those which people already have cannot be communicated to them by a definition*, but only by collecting together, for them to experience, suitable examples"
>
> (Skemp, 1971, p. 25).

Without the initial firsthand experience of the mathematical characteristics of the physical world which results in the personal construction of primary concepts, and subsequently without the personal construction of secondary mathematical concepts, there can be no understanding of higher order secondary mathematical concepts.

The development of constructivism as it has been applied to education has largely manifested itself in the areas of mathematics and science education. This is so for possibly two main reasons. The first is that these are prototypes of subject areas that deal with "objective" and "provable" knowledge, knowledge that was previously

conceived as being dictated purely by nature, rather than by the learner's previous understandings .

> According to this traditional view, scientific knowledge is constrained by, and largely shaped by, those aspects of the natural world that are the subject of study...If constructivism can take root in these disciplines, it can succeed anywhere, and this is one reason why the academic world has paid so much attention to mathematics and science...
>
> (Phillips, 2000, p. 5)

Another reason for the primacy of constructivism in science and mathematics is that Piaget, one of the main forerunners of constructivism, paid much attention to the child's understanding of mathematical and scientific concepts. Piaget's view of learning was extrapolated from the biological view of the organism's need for equilibrium. He saw this mechanism as being paralleled in the cognitive domain.

> Piaget (1952) viewed the human mind as a dynamic set of cognitive structures that help us make sense of what we perceive. These structures grow in intellectual complexity as we mature and as we interact with the world we come to know and as we gain experience. Through maturation and experience, the groundwork for new structures is laid.
>
> (Brooks & Brooks, 1999, p. 26)

According to Piaget, the active process of equilibration is made up of the processes of assimilation and accommodation, in which the individual assimilates information into her or his existing schema while accommodating those schema in accordance with the nature of the new information. This allows the individual to actively construct her or his own new knowledge while taking into account information received from the external, physical world.

One of the pioneers of the more contemporary work on constructivist theory is Ernst von Glasersfeld. Von Glasersfeld (1991, 1995) developed a theory of constructivism, termed radical constructivism, which holds that human beings, because they must necessarily know the external world through their own senses and perceptions, can never objectively know any external reality. Because the individual has no access to objective external reality, because that reality must always be perceived through the lens of the knower's own understandings, whatever external reality may exist is irrelevant to the way human beings see the world. Radical constructivism sees knowledge not as representing some objective reality that is inaccessible to our subjective selves, but rather the world as seen by individuals. He presents the basic principles of radical constructivism :

> 1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.
>
> 2. a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability.
>
> b. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality.
>
> (von Glasersfeld, 1990, pp. 22-23)

According to radical constructivism, rather than looking for truth, people test the viability of their constructions by comparing their own ideas with those of others and with the extent to which they "work" in their experiential world, and alight on "objective" knowledge by proving their ideas valid in their social milieu. He holds that the constraints on individuals' fantasy in the construction of their world are provided by their social settings and their interactions with the environment in which they are situated and which mediate their personal understandings.

> If one accepts the critique that knowledge cannot be shown to represent reality in some iconic way, as a picture of the world, then one is left with a more subjective construction of reality - subjective because one abandons the effort to factor the human subject out of the process. Although the constructivist is relativistic, that relativism is tempered by stability achieved by the individual in relation to his or her experience with others and the external world.
>
> (Confrey, 1995, p. 194)

Thus, von Glasersfeld shifts Piaget's focus on the individual and the physical environment to a focus on the individual and the social environment. But, in spite of this acknowledgment of the influence of social factors on the individual construction of knowledge, this influence is, for von Glasersfeld, a constraining one. The world does not exist as a positive model on which to base knowledge, but only as a check on our constructions when they are felt to be out of sync with the world as we experience it. Like the earlier constructivist writing, constructivism as thus presented continues to focus on individual, psychological processes. Nor does it look at the ways in which the individual's actions may affect her or his social and cultural milieu. As such it falls short of offering a perspective for looking at the interaction that occurs in classroom situations, whether those be primarily concerned with the development of mathematical or pedagogical understanding.

This shortcoming of radical constructivism is addressed by different strands of sociocultural theories of learning which propose the interdependence between the psychological development of the individual and the social context in which this development occurs. These include the symbolic interactionism of Mead (1934) and Blumer (1969), situated learning theory as developed by Lave (1988), the theory of Mediated Learning Experience (MLE) (Feuerstein, 1990; Kozulin & Presseisen, 1994) and Vygotskian theory (Wertsch, 1985; Cole, 1990). Of these, in the present work, I have looked mainly at the ways in which situated learning theory and MLE theory have affected my constructivist framework.

Situated Learning

Situated learning theory, as opposed to the views of constructivism mentioned above, holds that learning is not something that takes place in the isolated mind of the individual. Meaning, rather, must be socially negotiated. Learning is a holistic enterprise in which the individual modifies his or her behaviour through activity in contexts involving a whole range of contextual factors which affect and are affected by the individual. These factors include the needs and desires of the individual in particular situations, their way of perceiving the situation as determined by past experience and present expectations, other people involved in the interaction, and the physical setting within which the activity takes place.

> Learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world.
>
> (Lave & Wenger, 1991, p. 51)

Rogoff stresses the mutuality that exists between individuals and context.

> ...the individuals' efforts and sociocultural arrangements and involvement are inseparable, mutually embedded focuses of interest. Rather than examining context as an influence on human behaviour, I regard context as inseparable from human actions in cognitive events or activities.
>
> (Rogoff, 1990, p. 27)

Lave (1988), in particular, concentrates her attention on learning in real-life learning contexts and, as opposed to the traditional pejorative attitude toward everyday as opposed to academic learning, argues for the quality of the concrete, individual or unique learning that arises in those settings while rejecting the abstract, general or

universal learning of the classroom. Although the main difference between situated learning theory and radical constructivism would seem to be in the former's emphasis on social context, Lave & Wenger (1991) see a danger in the over-emphasis of the social that might be understood from the Vygotskian focus on the mediation of culture and its internalization by the child. In trying to find a middle road between concentration on the individual and concentration on social practice which "only seems to eclipse the person" (Lave & Wenger, 1991, p. 52), they suggest that seeing learning as increasing participation in communities of practice "suggests a very explicit focus on the person, but as 'person-in-the world', as a member of a sociocultural community" (ibid., p. 50).

According to situated learning theory there is a reflexive relationship between the objective forms and activity of the world and the individual's subjective and intersubjective understandings. Together they constitute "both the world and its experienced forms" (Lave & Wenger, 1991, p. 51).

> ...knowledge and learning will be found distributed throughout the complex structure of persons-acting-in-setting. They cannot be pinned down to the head of the individual or assigned to tasks or to external tools or to the environment, but lie instead in the relations among them.
>
> (Lave, 1993, p. 9)

Similarities with Constructivism

A comparison of situated learning theory and a psychological constructivist approach to learning reveals aspects in which they are similar, those in which they seem to complement each other, and certain aspects which are seen by some as being contradictory. The major similarities between them are their common rejection of a positivist view of teaching/learning in which knowledge is seen as a commodity which is transmitted as is from the teacher to the learner, but rather as something that is built and developed by learners; the corresponding importance they both attribute to the role of activity in the learning process; and the crucial role of motivation and interest on the part of the learner.

Complementary Aspects

In addition to these similarities, there are characteristics of situated learning that, although on first impression may seem at odds with constructivist theory, may also be seen as complementing and enriching it. When taken into account, these characteristics

may potentially contribute to the increased effectiveness of classroom teaching that was previously based on constructivist principles alone. Most obvious and unproblematic of these would be the increased relevance for the learner that is achieved when grounding learning in everyday situations and ensuring whole-person activity rather than cognitive involvement alone.

> To focus on whole-person activity rather than on thinking as separate from doing implies a negation of the conventional division between mind and body. This negation is also reflected in the claim that "cognition" is seamlessly distributed across persons, activity and setting. This in turn implies that thought (embodied and enacted) is situated in socially and culturally structured time and space. This object-world, viewed as partially constructed with persons-acting, is an essential aspect of activity. Its constitution is a matter of sociocultural order writ large.
>
> (Lave, 1988, p. 171)

Another view of situated learning theory easily assimilated into constructivist theory is the view of content as emerging from context and activity. This view has particular relevance for the teaching of mathematics according to constructivist premises, in which understanding is meant to emerge from the previous knowledge of the learner rather than resulting from the transmission of abstract, static knowledge from the teacher to the child.

Learning as Apprenticeship

Situated learning theorists focus their attention on apprenticeship as the framework which enables deliberate teaching of skills and understandings through participation in concrete real-life activities. In these situations the apprentice learns through observing the expert in practice and through carrying out those tasks that they are capable of with the gradually diminishing guidance of the expert. The metaphor of scaffolding has been used to characterize the role of the expert teacher in these interactions.

> The scaffolding metaphor makes explicit the role the teacher takes during the teaching-learning process, Here the teacher does more than simply organize resources (both human and physical) , in which interactions are predominantly procedural, or which encourage cognitive conflict. The teacher actively models or instructs and assists children based on shared understanding ...Learning is viewed, not as individual construction of knowledge, but rather through the joint construction of knowledge within historical and culturally defined contexts.
>
> (Fleer, 1990, p. 116)

Rogoff points to an additional feature of apprenticeship, the role that peers often play in the in the development of the individual apprentice.

> Furthermore, the apprenticeship model has the value of including more people than a single expert and a single novice; the apprenticeship system often involves a group of novices (peers) who serve as resources for one another in exploring the new domain and aiding and challenging one another. Among themselves, the novices are likely to differ usefully in expertise as well.
>
> (Rogoff, 1990, p. 39)

Two concepts related to the idea of the learner as apprentice have relevance for teacher education. One is the idea of legitimate peripheral participation (Lave & Wenger, 1991), in which a person in the role of apprentice is slowly and gradually initiated into a community of practice, moving from peripheral to full participation, while at the same time legitimately contributing to the ongoing work of the community.

If the classroom is considered a particular kind of community of practice, and its creation and development over time a process in which students' legitimate participation, first more peripheral and increasingly more central, both influence and are influenced by occurrences, then the concept of legitimate peripheral participation can be helpful in analyzing and expediting this development.

> ...learning is an aspect of changing participation in changing "communities of practice" everywhere. Wherever people engage for substantial periods of time, day by day, in doing things in which their ongoing activities are interdependent, learning is part of their changing participation in changing practices. This characterization fits schools as well as tailor shops. There are not distinguishable "Modes" of learning, from this perspective, because however educational enterprises differ, learning is a facet of the communities of which they are composed.
>
> (Lave, 1996, p. 150)

The second concept, cognitive apprenticeship (Collins et al, 1989), distinguishes between situations where the goal is the acquisition of observable, physical skills and those where the purpose of the interaction is the development of cognitive skills. In cognitive apprenticeship, as in traditional apprenticeship, learners are involved in complex problem-solving activities and tasks which learners are challenged to master while receiving support when necessary in the form of hints, questions and actual help from the teacher. The focus here, however, is on the expert processes used to develop

conceptual and factual knowledge rather than physical skills.

> It is this dual focus on expert processes and situated learning that we expect to help solve the educational problems of brittle skills and inert knowledge.
>
> (Collins et al, 1989, p. 457)

Because the processes that are involved in developing cognitive and metacognitive skills are not physically observable as is often the case in traditional apprenticeship, these skills must be externalized.

> Cognitive research, through such methods as protocol analysis, has begun to delineate the cognitive and metacognitive processes that comprise expertise, which heretofore were inaccessible. Cognitive apprenticeship teaching methods are designed to bring these tacit processes into the open, where students can observe, enact, and practice them with help from the teacher and from other students.
>
> (ibid., p. 458)

As will be shown later on in this chapter, the theory of Mediated Learning Experience (Feuerstein & Feuerstein, 1991) demonstrates ways in which the underlying aspects of learning situations may be made explicit through the use of mediation.

Attention to these parameters of situated learning would seem worthwhile for educators working within the constructivist tradition in that they have the potential to increase the connectedness, relevance and meaningfulness of learning

Possible Contradictions Between Constructivism and Sociocultural Theories

There is one characteristic of situated learning theory which may seem to be at odds with constructivist theory - the issue of learning transfer. In this Lave (1993) suggests what may be her most novel and, for myself, counter-intuitive contribution to learning theory. Conventional thinking assumes that the goal of learning is to produce abstract, general concepts that will serve the individual in many different areas and situations. The widespread attempt of schools to provide learners with this ready-made abstract knowledge is particularly noticeable in the teaching of mathematics. It would seem that even constructivists, who appreciate the necessity of building one's own knowledge, continue to view the abstraction and generalization of knowledge as one of the major goals of education:

> ...abstraction from and generalization across "contexts" are mechanisms that are supposed to produce decontextualized (valuable, general) knowledge... movement toward powerful (abstract, general) knowledge is movement away from engagement with the world, so that distance "frees" knowers from the particulars of time, place, and ongoing activity.
>
> (Lave, 1993, p. 23)

Contrary to this view, Lave holds that the most valued learning, that which is most meaningful to the learner and effective, is learning that takes place in the interactive relationships between whole people and their environment.

> Theories of situated activity do not separate action, thought, feeling, and value and their collective, cultural-historic forms of located, interested, conflictual, meaningful activity. Traditional cognitive theory is "distanced from experience" and divides the learning mind from the world. This "release" from the narrow confines of body and immediate experience is rejected on varied grounds...in favor of more complex relations between person and world. The idea of learning as cognitive acquisition - whether of facts, knowledge, problem-solving strategies, or metacognitve skills - seems to dissolve when learning is conceived of as the construction of present versions of past experience for several persons acting together...
>
> (Lave, 1993, pp. 7-8)

In order to clarify this idea of situated knowledge, Brown, Collins and Duguid (1989) compare conceptual knowledge to a set of tools. Just as it is possible to acquire a tool without having any idea of how to use it, so may a student learn abstract knowledge such as mathematical algorithms or dictionary definitions without being able to put them to use. It is only through participation in the particular culture which devised the tool that the individual can know its use, and only through repeated use that he or she can understand how it is used and become able to do so ably and efficiently.

> Appropriate use [of conceptual tools] is not simply a function of the abstract concept alone. It is a function of the culture and the activities in which the concept has been developed. Just as carpenters and cabinet makers use chisels differently, so physicists and engineers use mathematical formulae differently. Activity, concept, and culture are interdependent. No one can be totally understood without the other two.
>
> (Brown, Collins & Duguid, 1989, p. 33)

Collins et al (1989) seem to have found ways of reconciling this apparent contradiction between views of learning in the distinction they make between cognitive apprenticeship and traditional forms of apprenticeship.

> Traditional apprenticeship emphasizes teaching skills in the context of their use. We propose that cognitive apprenticeship should extend situated learning to diverse settings so that students learn how to apply their skills in varied contexts. Moreover, the abstract principles underlying the application of knowledge and skills in different settings should be articulated as fully as possible by the teacher, whenever they arise in different contexts.
>
> (Collins, Brown & Newman, 1989, p. 459)

In this way Collins et al. seem to have neutralized the perceived contradiction between the situated nature of learning and the possibility and desirability of the existence of learning transfer. Their view smooths the way for the integration of situated learning and constructivism, and also makes room for the parameter of Mediated Learning Experience that Feuerstein calls "transcendence", to be discussed below.

Thus, the importance for constructivists of this integration would seem to lie in the demonstrated effectiveness of learning within relevant, meaningful and involving contexts. Lave discusses the effectiveness of learning when it is carried out in real-life settings:

> It is difficult, when looking closely at everyday activity...to avoid the conclusions that learning is ubiquitous in ongoing activity, though often not recognized as such.
>
> (Lave, 1993, p. 5)

Further justification for taking advantage of situated learning theory in teacher training lies in the evidence that suggests that even teacher education courses run along constructivist lines, have not produced the desired results in students' future teaching (Klein, 2001). This lack of transfer of learning may be at least partly a result of the attempt of many college courses to teach pedagogy without the necessary grounding in real life school classroom situations. Situated learning theory, whether incorporated into a constructivist framework in all its aspects or considered along-side as a helpful conceptual framework in its own right, would seem to have the potential to somewhat rectify this situation. With this in mind I felt it necessary to allow the insights of situated learning theory to inform my practice in my action research.

Social Constructivism

The social constructivist or emergent approach in mathematics education came out of the attempt to integrate the sociological approaches towards learning of symbolic interactionism (Mead, 1934; Blumer, 1969), Vygotskian theory, (1933); and situated learning (Lave, 1988) with the psychological cognitive approach of radical constructivism.

> Viewing individual activity as an act of participation in a system of practices that are themselves evolving... is inherent in the central organizing metaphor of the situated perspective and serves to differentiate it from the cognitive perspective...
>
> (Cobb & Bowers, 1999, p. 8)

For Cobb and Yackel (Cobb & Yackel, 1998; Cobb & Bowers, 1999) , the need for a wider explanatory framework arose from research into the mathematics education of young children which highlighted social, in addition to psychological, factors involved in their mathematical development. A long-term developmental research project into mathematics education in the lower elementary grades originally intended to focus on individual children's conceptual development in their learning of mathematics. The project extended to the classroom context the one-on-one constructivist teaching experiment where the researcher, working with individual children, attempts to influence the ways in which children construct their conceptual understandings. Although from the outset Cobb and Yackel were influenced by the interactionist perspective of Bauersfeld (1988) and others, and therefore emphasized the mathematical communication that occurs between student and teacher in the course of a mathematics lesson, the role of social interaction as they conceived it at that time was as a catalyst for the individual development of mathematical meaning. This view was similar to that of researchers who based their work on Piaget's developmental psychology. Voigt characterizes this research as follows:

> ...they explore social interactions between children and conflicts between the children's perspectives as conditions of mental reorganizations. However they conceptualize social interaction as an external variable with regard to developmental processes inside the learner.
>
> (Voigt, 1992, p. 7)

Cobb and Yackel describe their original position in a similar way:

> With hindsight, it is apparent that the relation between social interaction and children's mathematical development implicit in this approach was neo-Piagetian. We assumed that conflicts in individual students' mathematical interpretations might give rise to internal cognitive conflicts , and we assumed that this would precipitate mathematical learning...In this account, social interaction was viewed as a catalyst for otherwise autonomous psychological development because it influenced the process of mathematical development but not its products, increasingly sophisticated mathematical ways of knowing.
>
> (Cobb & Yackel, 1996, p. 177)

The joint efforts of these former radical constructivists and those of sociocultural theorists have resulted in a theory of learning that allows the different aspects of learning to be incorporated into a more holistic understanding of this complex and multifaceted process (Cobb, 1989).

> In this approach (the emergent perspective), individual thought and social cultural processes are considered to be reflexively related with neither attributed absolute priority over the other....However, adherents to the emergent perspective treat the linkage between social and cultural processes and individual psychological processes as an indirect one (cf. Saxe & Bermudez , 1996; Voigt, 1994). Thus, they might talk of students' participation in particular cultural practices as enabling and constraining their mathematical development, but not determining it...The extent to which either a psychological or a social analysis is brought to the fore in any particular situation is a pragmatic issue that reflects the purposes at hand.
>
> (Cobb et al, 1997, p. 152)

Through their research Cobb and Yackel saw how social and mathematical norms emerged through mathematical communication that occurred during the lesson. These norms were mutually constituted by the teacher and the students, resulting in the formation of a classroom mathematical community. Yackel characterizes this relationship:

> ...we consider the relationship between the sociomathematical norms that are constituted in the classroom, a sociological construct, and the individual participants' mathematical beliefs and values, the psychological correlate. To say that these two sets of constructs are reflexively related means even more than that they are mutually enabling and constraining. It means that one literally does not exist without the other. For example, as children give explanations they deem viable, they are not only participating in the taken-as-shared normative understandings of what constitutes acceptability, they

> are also contributing to the ongoing negotiation of what is taken as normative. Thus, their (individual) mathematical beliefs and values develop concomitantly with the classroom sociomathematical norms"
>
> (Yackel, 1997, p. 4).

Parallels Between the Teaching of Mathematics and the Teaching of Mathematical Pedagogy

In my work with education students I began to see parallels between the sociomathematical norms that may be developed in a mathematics class and sociopedagogical norms that are relevant for teacher education. In the prevailing forms of mathematics teacher education discussed above, in which pedagogical discussions are based on tasks aimed at developing *mathematical* knowledge, the development of sociomathematical norms may continue to be the focus (Simon, 1995, Civil, 1993). In teacher education courses it would seem necessary to provide high-level dynamic knowledge construction activities (Dunlap & Grabinger, 1996) aimed at developing *pedagogical* knowledge that would lead to the construction of sociopedagogical, in addition to sociomathematical, norms. These activities would be such that the students would deal with "ambiguous information, authentic ill-structured problems; controversy and argumentation; and judgments and decision making" (ibid., p. 76) in the realm of pedagogy.

Whose Pedagogical Norms?

There are a number of ways in which these parallels might be helpful in guiding teacher education. One is brought to mind by Klein's (2001) account of the difficulties she encountered when, in teaching a pre-service mathematics education course based on constructivist theory, her students reported feeling disempowered in that they felt they were expected to conform to the instructor's views of teaching. This was one of the dilemmas that concerned me in my own teaching as well - is it not problematic to have education students use their past experience of mathematics education, which is normally so at odds with alternative, non-traditional practice, as a basis for solving pedagogical problems? The tensions that might arise from a clash between the two have, as shown by Klein, the potential to create a power struggle between the teacher, who believes deeply in a constructivist theory of education, and the students. There are two connected potential remedies to this situation. One lies in a view of the teaching of pedagogy that is more in line with constructivist theory, where the construction of

the students' own pedagogical knowledge, and the accompanying diversity in people's thinking that manifests itself, are both sanctioned and highly valued. While this may lead to a prolonged development of hoped-for sociopedagogical norms, it is possible that with appropriate activities and scaffolding techniques, these may eventually develop in a more legitimate and empowering manner. The other is in the instructor's attempt to provide evidence for the legitimacy of his or her educational approach (McNiff, 2003, personal communication). Just as in mathematics education we encourage students to justify their solution strategies, so in the teaching of pedagogy it is necessary to provide evidence of good educational practice. The development of classroom norms is a collaborative effort of both the teacher and the students. While it would seem illegitimate to force our views on our students, so is it misbegotten to make no attempt to influence them at all (Buchmann, 1986). As a teacher I must do and say things that will stimulate their thinking and provide at least some of the raw material from which they will eventually derive their own beliefs. This parallel with work in mathematics education is further illustrated in the following section.

A Reflexive Relationship

Cobb and Yackel's view of the inquiry approach to mathematics education, as opposed to the school mathematics tradition, may be seen to be relevant for this kind of "inquiry pedagogy" as well. They hold that inquiry mathematics deals with truths, while traditional teaching of mathematics deals with instructions. The parallel can easily be seen between traditional mathematics instruction which deals in rules and techniques and didactics courses whose focus is on teaching techniques, helpful hints and activity ideas. The truths that can emerge from inquiry mathematics are seen as being both constructed and normative:

> At first glance, this view that truths are normative might seem questionable given that truths appear to tell us how the world is, not how it ought to be. However, as Much and Shweder and numerous others have noted, individuals are typically challenged by other members of their community when their actions transgress a currently accepted truth...in this view, members of a community such as the teacher and students in a classroom interactively constitute the truths that tell them how the world is or ought to be, and these truths constrain their individual activities.
>
> (Cobb & Yackel. 1998, p. 163)

Similarly, pedagogical truths can be interactively constituted in the didactics class, and the ways that this might happen are similar to the processes in mathematics classes:

> (by) explaining and justifying solutions, attempting to make sense of

> explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations in which a conflict in interpretations or solutions has become apparent".
>
> (ibid., p. 167)

Bauersfeld (1988), from an interactionist perspective, discusses the contribution that individual students make in the emergence of the mathematics classroom culture. Individuals do not just rediscover existing truths, but rather interact reflexively with the culture that is influenced in turn by them.

> Both teacher and student contribute to the classroom processes. It is a jointly emerging "reality" rather than a systematic proceeding produced or caused by independent subjects' actions.
>
> (Bauersfeld, 1988., p. 29)

Similar to the constraints put on the individual construction of knowledge, the influence that individuals have on the community is not arbitrary. Within the classroom, as within the wider professional mathematics community, the community sets standards of justification and validation that must be met.

> Neither the teacher nor the student alone can be blamed for the course of the process; they are not aware of the covert pattern of their joint actions. As a product [of] social interaction, the patterns develop from the mutual reflexive expectations of the actors and their related moves, from the implicit "obligations for action", which are typical for the institutionalized educational processes, and from the teacher's and the students' routines as acquired across many shared classroom experiences.
>
> (ibid., p. 37)

Within the college mathematics education classroom, therefore, there may be seen to be an emerging pedagogical reality just as there is an "emerging mathematical reality" (Cobb & Yackel, 1998) in the mathematics classroom.

In spite of the value attributed to this joint construction activity, and the respect with which student attitudes and understandings are held, Cobb & Yackel show that teachers unconsciously guide and direct the understanding of their students. Teacher reactions to the contributions of the students play an important part in determining what is valued in the classroom community.

> (Teachers') reactions to students' solutions frequently functioned as implicit evaluations that enabled students to infer which aspects of their mathematical activity were particularly valued. As Voigt (1993) notes, these implicit judgments made it possible for students to become aware of more developmentally advanced forms of mathematical activity while leaving it to

> the students to take up the intellectual challenge. Students could therefore develop a sense of the teacher's expectations for their mathematical learning without feeling obliged to imitate solutions that might be beyond their current conceptual possibilities. From our perspective as observers, we can in fact view this as one of the ways in which the teachers attempt to cope with a tension inherent in teaching, that between mathematical learning viewed as a process of active individual construction, and mathematical learning viewed as a process of enculturation into the mathematical practices of the wider society.
>
> (Cobb & Yackel, 1998, pp. 169-170)

Teacher education instructors, as the representatives of a professional view of mathematics education, may similarly influence the development of pedagogical norms without forcing their views on their students.

In my college mathematics education course my primary concerns needed to be both the development of this pedagogical reality and the unfolding of a new mathematical reality. Noel (2000) relates to the necessity of taking advantage of this approach at all age levels.

> If one thinks of social constructivist models as useful frameworks to conceptualize how we learn, no matter what our age, then we as educators need to work towards replacing the transmission orientation of university classrooms with opportunities for active engagement and interaction with a variety of environments and individuals (Miller, 1988).
>
> (Noel, 2000, p. 17)

The Community of Reference

This leads to a discussion of an additional concern of both mathematics education and teacher education: the intellectual community that serves as the point of reference for the study of the discipline. The classroom teacher may be considered ideally to be a representative of the particular discipline being taught. In mathematics, some would see that discipline to be that of the professional mathematician, others the community of mathematically proficient adults (Burton, 1992). As is too often seen, many elementary school teachers are representatives of neither. A socioconstructivist approach to the teaching of mathematics in teacher education, in which future teachers are introduced into the world of authentic mathematics activity, can begin to rectify that situation.

There is a similar necessity in regard to the wider pedagogical community of mathematics educators which must inform the pedagogical learning and understanding of education students. Many education students enter teachers college believing that

they already know what teaching, and specifically mathematics teaching, is all about. In their eyes, the only reason they are in education school is to receive a few techniques for achieving this taken-for -granted kind of learning in their students. Buchmann (1986) addresses the problem of the view that teachers must teach in ways that come naturally and in accordance with their personal inclinations and preferences,

> What teachers do is neither natural nor necessary but based on choice. Since choice may harden into custom or dissipate into whim, one asks for justification; it is a way of ensuring that teaching will periodically pass muster. In justifying their actions, people give reasons. For teachers, personal reasons can be appropriate when explaining a given action to others, but they carry less weight when considering the wisdom of an action or decision.
>
> (Buchmann, 1986, p. 529)

Solomon makes a similar argument:

> Personal knowing is notoriously precarious and subject to brain-washing, unless socially reinforced. The sociologists of knowledge, from Mead to Schutz and Vygotski have all shown that our common ways of communicating do more...than pass on messages. They are sophisticated efforts, not so much to describe things as they 'really' are, but to share meanings with others. This is done not through a comparison of experiences but through the operation of intersubjectivity, as a process...
>
> ...Intersubjectvity is a process of interchange, a dynamic during which meanings are chosen from a repertoire of language tools into which the speaker/hearers have been socialised.
>
> (Solomon, 1994, p. 15)

Just as in mathematics lessons where providing acceptable justification depends on referring to experientially real mathematical objects (Yackel & Cobb, 1996), determined by the logico-mathematical nature of mathematical knowledge and goals of mathematical activity and accepted as such by the members of the classroom community, so in teacher education classes acceptable reasons for particular teaching behaviours depend on the experientially real role obligations of teachers as defined by the professional community.

> Providing acceptable justification requires the existence of a community to both set standards for adequacy and determine a set of rules for guidance. The role obligations of teachers as members of such a community forge bonds that not only ensure compliance but generate effort and involvement.
>
> (Solomon, 1994, p. 15)

Buchmann lists three aspects of teacher obligations: they do not depend on any particular individuals; they are not dependent on individual preference; they are determined by the subject matter that is being taught and learned:

> In schools, teachers are supposed to help students participate in "the community of subject matter" whose objective contents of thought and experience - systems, theories, ideas - are impersonal because they are distinct from the people who learn or discuss them. They are, to some extent, independent of time and place.
>
> (ibid., p. 531)

Buchmann's use of the term "objective" is reminiscent of the traditional assumption of the objectivity of knowledge in general and mathematical knowledge in particular and reintroduces the argument between a positivist view of knowledge and a radical constructivist view. For the purposes of the discussion here, however, it is equally possible to understand objectivity in the radical constructivist sense of socially agreed upon knowledge. It would seem, therefore, that in both subject matter domains, mathematics and pedagogy, there are widely accepted views of professional and/or educated behaviour that must guide the education of novices in each field.

The conflicts and tensions that arise from needing to simultaneously take into account the students, the teacher, and the wider mathematics education community, to name only a few of the factors that influence pre-service mathematics education, have been the impetus for this research into the teaching of mathematical pedagogy. The work that has been done on mathematics education has provided a framework which has guided me in my search for ways to reduce these tensions, thus becoming more effective in my teaching.

THE THEORIES OF REUVEN FEUERSTEIN

The theory of social constructivism, with both its psychological and sociological elements, discusses the way in which people learn, not the way they should be taught (Simon, 1995) As discussed above, models of classroom practice in accordance with social constructivist theory have been developed for the teaching of mathematics both to children and to teachers. Models have also been developed for the teaching of mathematics pedagogy (Simon, 1995, Korthagen & Kessels, 1999; Carlson, 1999; Dunlap & Grabinger, 1996; Carpenter, 1992; Klein, 2001). Feuerstein's theory of Mediated Learning Experience (MLE), similar in many ways to Vygotskian theory,

while theorizing about learning, also provides a detailed, practical way of looking at learning and teaching which can aid teachers at all levels of education in ensuring that learning takes place for their students and in developing their own teaching abilities and style.

MLE theory may be seen to be both constructivist in the Piagetian sense and sociocultural. Feuerstein, a student of Piaget, concurs with him in seeing the central role that the individual plays in the construction of his or her knowledge.

> The formula proposed by the Piagetian school, known as "stimulus, organism, response" (SOR), ascribes to the organism, its level of maturation, and its stage of development, an important role in the registration and active elaboration of stimuli...Piaget, emphasizing the characteristics of organisms and their maturation, introduced the notion of activity as no less responsible for the development of intelligences than the properties of the stimuli to which the organism is exposed.
>
> (Feuerstein & Feuerstein, 1991, p. 9)

Feuerstein holds, in fact, that most knowledge is gained through this direct contact of the individual with the environment. He notes, however, that this mode of interaction cannot account for the differences between individuals in the extent to which they are able to change (learn) through contact with this external environment. These differences Feuerstein attributes to the culturally determined intentions and actions of adults or more experienced peers who mediate the environment for the learner. Although mediation is seen to be a circumscribed interaction between a single, intentioned adult (or more experienced peer) and learner or learners, the mediator is seen to be a representative of the wider culture, mediating that culture for the learner.

Intelligence as a Changing Entity

Feuerstein (1998) begins by offering his own definition of intelligence. As opposed to the conventional view of intelligence as an entity that is fixed and can therefore be measured, he sees intelligence as

> ...energy, an impulse (instinct?) , the need to change in order to adapt to any situation and to deal with it in an effective way.
>
> (Feuerstein 1998 p. 23, translated from the Hebrew)

This energy results in the modification of thinking structures and patterns of reaction - in

other words, in an increase in intelligence. It is not sufficient in his eyes, however, to consider this as merely a proposition:

> I will begin my response in an expression of belief, and that I do purposefully, in spite of the fact that as a man of science I am expected to steer clear of such 'unscientific' expressions: we **believe** that the humans are modifiable beings who are not only capable of changing, but are capable of changing themselves according to their own will and their own decisions.
>
> (Feuerstein, 1998, p. 13)

Feuerstein sees this expression of belief in the ability of human beings to change as a necessary precondition for any teacher working with children or with adults. If teachers do not believe that it is possible for the learner to learn, develop and change, they can easily produce the opposite effect of what they might theoretically intend, to help their students learn. In the area of mathematics education, for instance, it is not uncommon to hear teachers characterize their students as being mathematically-minded or not. Those who are not are often expected not to understand, and are treated differently as a result of these different expectations.

This view of intelligence as a changing entity is the guiding principle behind Feuerstein's theories of Structural Cognitive Modifiability and Mediated Learning Experience (MLE).

Structural Cognitive Modifiability and Mediated Learning Experience

According to the theory of Structural Cognitive Modifiability, human beings are capable of undergoing structural cognitive change - when change occurs in one particular area of understanding, it affects the whole of the individual's cognitive makeup. Feuerstein distinguishes between learning that is simply the addition of bits of information that do not affect the organism in an essential way, and learning as a result of which the cognitive structure of the individual is changed. According to Feuerstein (1990), there are four dimensions of structural modifiability:

*it is preserved over time

*it is stable in the face of changing conditions

*it is generalized over and above the specific incident

*it continues to change and to create further change.

In a situation where the individual's thinking is structurally changed by a particular

learning experience, he or she will be able to take advantage of the tools acquired on that occasion when encountering new and different circumstances. The individual acquires not only new information, but also new cognitive structures which open up new areas not previously part of her of his existing repertoire and skills.

According to Feuerstein, however, left on their own, individuals are often unable to take advantage of many of the learning opportunities available to them in their environment. As discussed above, this view is in opposition to Piagetian constructivism, which holds that the only modality of learning is through direct experience with the environment:

> For Piaget, learning occurs in an unassisted interaction between the child's mental schemas and the objects of the external world ... The only requirement is for the learning milieu to be sufficiently rich so that the child has enough objects and processes with which to practise his or her schemas."
>
> (Kozulin, 1994, p. 270)

Mediated Learning Experience is the name given by Feuerstein to portray interactions between mediator and learner which result in structural cognitive modifiability. He holds that the adult's mediation (or that of a more experienced peer), which directs the learner's attention to the learning possibilities inherent in a situation and in different ways prepares the preconditions necessary for learning, is of major importance in order for significant learning to transpire.

> MLE is defined as a quality of human-environment interaction that results from the changes introduced in this interaction by a human mediator who imposes him/herself between the receiving organism and the sources of stimuli. The mediator selects, organizes, and schedules the stimuli, changing their amplitude, frequency and saliency; and turns them into powerful determinants of behaviour instead of randomized stimuli whose occurrence, registration and effects may be purely probabilistic.
>
> (Feuerstein, 1990, p. 78)

It is the unique contribution of MLE that it renders the learner more capable of learning in the future - it increases the adaptability of the organism to its environment, rendering it more modifiable when faced with environmental changes.

> The change produced by MLE has not only been in the realm of learned content, but in the learning structure, in the propensity for learning, and in the growing capacity of the organism...to benefit from exposure to learning situations.
>
> (ibid., p. 76)

Although knowledge may be gained through direct experience with the environment, this propensity for learning cannot.

> We argue that the human being can acquire much knowledge also without mediators, seeing as he or she is born into the world equipped with the skills that enable continued development and progress through direct stimuli. But the ability to change and the characteristic flexibility of the human are acquired through mediated learning experiences.
>
> (Feuerstein, 1999, pp. 74-75)

Feuerstein holds, therefore, that it is the responsibility of adults, including teachers, to mediate the environment for the learner. Teachers must become aware of the critical influence that they necessarily have on the development of their students' intelligence. It is their role and responsibility to orchestrate learning experiences that will provide their students with the best opportunity to develop their own thinking. Teachers who are not aware of their influence can generate classroom experiences that either do not support this development or result in the actual obstruction of further development. Reminiscent of Dewey (1963), Feuerstein asserts that experience can have the result of blocking further learning rather than stimulating it:

> Any experience is miseducative that has the effect of arresting or distorting the growth of further experience.
>
> (Dewey, 1963, p. 26)

Feuerstein emphasizes the importance of this information for the teacher:

> This knowledge is needed by the teacher in order for him to understand his responsibility - this organism is changing at his fingertips.
>
> (Feuerstein, 1999).

Equipped with this understanding, both teachers and teacher educators can no longer accept with equanimity teaching practices that result in the decreased ability of the learner to learn.

> ...the centrality of a human mediator in the teaching process should be emphasized and, as a result, appropriate changes instituted in teacher preparation. Teachers should be ready to work on the enhancement of every student's learning potential and zone of proximal development, rather than on just covering information required by the curriculum.
>
> (Kozulin & Presseisen, 1995, p. 74)

It is important to note that for Feuerstein the goal of the learning process is the development in the learner of the ability to learn autonomously. An individual who has

received sufficient mediation in the past becomes capable of mediating for him or herself in the future. The emphasis placed on the generalization of knowledge and on its transfer between domains will enable the learner to learn new fields of knowledge in an independent manner as well. Connected with this is the proposition that along with producing modifiability, MLE results in more flexible and diverse thinking:

> ...one of the most important characteristics unique to humanity is its considerable plasticity and flexibility, making the human capable of modifying and diversifying his cognitive structure in a radical way which will affect his capacity to learn to adapt to more complex and unfamiliar situations.
>
> (Feuerstein & Feuerstein, 1991, p. 12)

The relevance of these hypotheses for mathematics education may be seen in the importance that the use of MLE may have both for a population whose experience of mathematics has often been miseducative, and for the initial mathematics experiences of children in their first years of schooling.

Because of the central role in my research of MLE theory and its practical applications as outlined by Feuerstein, I will attempt to portray those aspects that are most relevant to my work in a sufficiently detailed way, while showing how they can be useful for a social constructivist theoretical framework.

The Parameters of MLE

In order to understand and operationalize the theory of MLE, Feuerstein has articulated 12 parameters of mediated learning experiences that point to the qualities of those interactions that make learning interactions both effective and significant. These parameters offer the mediator a way to plan, carry out, and evaluate their teaching. Of these, the first three, intentionality and reciprocity, transcendence, and mediation of meaning, are considered the necessary conditions for any mediated learning experience - without these an interaction will not be MLE. Ben Hur (2000) characterizes the work of teachers who act as mediators using these three criteria of MLE:

> Teachers who act as mediators *intend* to structurally change, cultivate, and crystallize specific student cognitive behaviors and/or concepts; they target student cognitive behaviors and/or concepts whose values *transcend* the specific time, place, context and content they use in teaching; and they form learning experiences that are imbued with *meaning*.
>
> (Ben Hur, 2000, p. 44)

Here I will briefly analyze these first three criteria as they relate to the work of teachers, whether they be working in schools with children or in higher education with adults.

Intentionality and Reciprocity

The importance of interactions that can be classified as MLE lies in the focus on the effects they have on the learner. Unlike the all-too-common notion of teaching, for Feuerstein it is not enough that the teacher teaches - "covering the material" will likely not result in any significant effect on the learner.

> We can communicate units of information to the child. We can give him instructions. We can tell him a story. We can offer him food, sing him a song or talk about the clouds without necessarily mediating to him ...In order to turn this interaction into a mediated learning experience, we have to give the interaction a special quality, one that is necessary in order to affect the child's cognitive system and produce a higher level of modifiability in him.
>
> (Feuerstein, Rand, Ryders, 1988, p. 60)

The criterion of intentionality and reciprocity relates to this quality of MLE . In order to be said to be mediating, the mediator must mediate his or her intention to do so and, additionally, to ensure reciprocity on the part of the learner. Reciprocity is that response that ensures both the learner's cooperation in the learning activity and his or her cognitive structural change. From Feuerstein's point of view, it is not the acquisition of subject matter that is the primary goal of the learning interaction, but rather the ability of the learner to take advantage of learning opportunities in the future. The teacher should be aware of the primacy of this goal and mediate this intentionality so that the learner, too, will be aware of it.

> In a learning situation the child should realize that the real objective of learning activity is not a particular task or puzzle, but the child's own thinking.
>
> (Kozulin & Presseisen, 1995, p. 70)

In an interaction characterized by intentionality and reciprocity , the mediator takes into account the physical, mental and emotional state of the learner as well as the attributes of the material to be learned, and accommodates these to each other. At the same time, the learner responds by taking into account the intentions of the mediator and the attributes of the material as they are perceived. The learner is aware of the fact that his or her needs and desires are being taken into account and responded to and is therefore more likely to feel empowered and willing to cooperate with the mediator. Tzuriel discusses the empowering aspect of intentionality and reciprocity. Relating to mother-

child interactions, he notes:

> The reciprocity aspect is characterized as a process in which the mother and the child are mutually and alternatively responding to one another. The reciprocity aspect is conceptualized as essential for the development of basic feelings of competence and self-determination. Interactions imbued with reciprocal intentionality assist the child in realizing that his actions determine other people's behaviors and foster his organismic belief that he is the agent of change.
>
> (Tzuriel, 1991, p. 97)

This affective quality of reciprocity is suggested by Klein as well:

> An environment that reacts to the child enables him to learn a basic principle that is likely to accompany him during his whole life, and that is that people listen to him, react to him or in other words, that it is worthwhile to act. A child that lives in a world where everything happens without any connection to his own behaviour...learns that there is no point in trying to act because in any case everything happens regardless of what he does.
>
> (Klein, 1985, p. 41, translated from the Hebrew)

This reflexive relationship between the mediator and the mediatee brings to mind the view of situated learning theory that locates learning in the interaction between the learner and the context (Lave, 1988; Lave & Wenger, 1991, Rogoff, 1990). Rogoff (p. 17) looks at the intentionality of both the child and the caregiver, where the child might purposely attend to the acts of a trusted guide in order to benefit from those acts. This similarity is strengthened by the way in which Feuerstein sees change occurring in all participants in the interaction. In an interaction characterized by intentionality and reciprocity there are three factors involved - the mediator, the mediatee, and the object to be mediated. The object, or the material to be learned, is changed by the mediator so that it will become more salient to the learner - it may be physically presented in such a way that the learner will notice it, by positioning the thing itself directly in front of the learner, by bringing attractive illustrations of the real phenomenon, by highlighting a particularly important passage in a text through bold type, through repetition, or through reading it louder and more deliberately than the rest of the text.

The mediatee is changed in two ways. First, before any attempt at mediating the content is made, his or her attention must be gained. In order to do this the mediator can simply point out the importance of the learner's attention at that particular moment, or s/he can say something surprising, make a joke, or change his or her tone of voice - thus ensuring the learner's fullest attention to the object to be studied. Second, the learner, as a result of the interaction, undergoes cognitive structural change, change that will affect his or her ability to continue to learn in the future.

Finally, the mediator/teacher is affected by the interaction as well. S/he must learn from the learner as well as from the particular subject matter with which the learner is faced. S/he changes her actions, s/he changes her strategies, s/he changes her ways of thinking - all these are necessary preconditions if s/he is to mediate for the learner in the most effective way possible.

This situated view of the mediator's learning emphasizes the importance of MLE for the mediator him or herself. Interaction with the learner and the material results in his or her continual learning and development. It provides at least one of the ways by which professional development can continue as a life-long process initiated by the practitioner in the framework of day-to-day work with students.

Transcendence

Transcendence may be seen as the mediator's attempt to extend a current and specific learning situation to diverse virtual contexts so that the learner will learn when and how it is possible to apply presently-learned skills and concepts in varied contexts. Feuerstein, like Collins et al. (1989) in their construct of cognitive apprenticeship, emphasizes the importance of both lateral and vertical transfer, lateral being the transfer of skills or concepts across situations, while vertical transfer, occurring between lower-level and higher-level concepts or skills, involves their explicitly articulated abstraction.

In mediating transcendence the mediator attempts to distance the learner from the immediate situation that is being experienced. When an individual is interested only in the satisfaction of immediate needs, actions are carried out in response to the way the situation originally presents itself, with little or no consideration of alternatives and therefore offering no opportunity to engage in significant reflection or problem solving. The mediation of transcendence widens the scope of the interaction to areas that are suited to the fulfillment of more remote goals. An adult who grabs a child about to run into a busy street achieves the immediate goal of the interaction. But it is only through his or her accompanying explanation of the reasons for the act that the child will a) begin to understand that it is always necessary to look before running into the street, and b) begin to understand that there are reasons behind encounters with adults in their daily lives that might otherwise seem arbitrary. A system of expectations will develop in regard to all phenomena that are met in the child's world - things generally do not just happen in random and unpredictable ways. There are reasons behind most

occurrences, and these reasons are often, to some extent at least, accessible to the child who questions and explores these phenomena.

> Structural cognitive change in the child is made possible only by transcending the satisfaction of his immediate needs. These changes in his thinking are expressed, for example, in the need and expectation of the child to find additional information that is connected with what is happening in the immediate present. In other words, the child's repeated experience of the satisfaction of immediate needs which is accompanied by verbal or non-verbal mediation of content and meaning that are not necessarily part of the satisfaction of the original need - that is what creates in the child the expectation that every experience does not stand alone in space, that experiences happen in context with other things, that there are reasons and results.
>
> (Klein, 1985, p. 44 - translated from the Hebrew)

Achieving the major goal of MLE, increasing the flexibility of the learner's cognitive system, can only be done if the opportunity presents itself to stand back, review the situation from a distance, weigh possible alternative solutions, and choose that which will best achieve a future goal. This process results in the identification of underlying principles which can in turn be transferred to other situations and tasks. Because most of our interactions with the world are direct interactions, without the benefit of mediation from another human being, the transcendent nature of MLE is of prime importance.

Feuerstein also calls transcendence the "humanizing factor" - it is what distinguishes between human functioning and what seems to be the functioning of other animals. (Feuerstein, 1998, p. 52). This factor is expressed in the human need to look for cause and effect, but it is also evident in the collective and cultural nature and needs of human beings. The values and beliefs of a particular culture are mediated through the significance given to particular occurrences over and beyond the immediate goals of the interaction.

An example of the mediation of transcendence in mathematics education, for instance, may occur when a mistake is not seen as a negative phenomenon, but rather as an opportunity to understand why the mistake was made, thus enabling further learning. In an MLE interaction, the mistake would not only be discussed and understood, but the importance of errors and their positive contribution to the learning process is mediated. This interaction can help to establish a different communal norm in regard to the making of mistakes, the learner is personally involved in a developing community of practice, and the result is the learner's structural cognitive change.:

> Transcendence, established by cultural imperatives, preferences, or styles, enriches the mediational interaction with components of time, space, succession, order, culture, and other dimensions that have importance to the development of the child's cognitive structures.
>
> (Feuerstein & Rand, 1997, p. 65)

Mediation of Meaning

In the mediation of meaning, the mediator invests stimuli, events or information with "personalized meaning", meaning which is derived from his or her own background and understanding. Feuerstein sees the mediation of meaning as being the "energetic factor" of MLE. Without a sense of the importance of the material being learned, without knowing the reason that this material has been chosen for study, there is little likelihood that the learner will choose to invest the effort necessary to learn. This involvement of the learner in the learning process not only ensures the receptiveness of the learner to the world, but also the fact that he or she "is engaged in a mutual interaction with it" (Feuerstein & Rand, 1997, p. 66).

An important effect of this criterion of MLE is that learners for whom meaning has been mediated will come to assume that there must be reasons for learning particular subject matter. As such, they will become accustomed to look for meaning in other situations as well, and finding meaning will become a necessity for them in any learning situation. As a result of this propensity to ask "why" and "what for", these learners will begin to look critically at incoming information rather than passively accepting the prescripts of accepted authority figures. The mediation of meaning

> reflects the need systems of the mediators as a determinant of their intention and their perception of the goals for the future that they set for themselves and their progeny or their mediatees.
>
> (Feuerstein, 1990, p. 98)

In the case of teachers of mathematics, this questioning of content and the search for justification of what and how particular content is being taught may change the very essence of their teaching. This is the case both in their role as receivers of curriculum and in their role as mediators of meaning for their students. It is often the case that teachers themselves are not aware of any particular reason for teaching a subject other than the fact that it is either a curriculum requirement or that it is simply taken as a societal given - as is the continued teaching of the long-division algorithm in spite of its

having been deleted from the mathematics curriculum in many places in the world. Even when teachers understand the significance of what they are teaching, however, and have chosen to teach in a certain way for particular educational reasons, it is often the case that they do not think to mediate this meaning to their pupils. From the point of view of the children, the material is "studied" because that is what the teacher has decided must be. Doubts regarding the utility of a particular subject is often voiced by children outside of the classroom, not directly to the teacher, who might otherwise actually take up the challenge and attempt an answer. The necessity of mediating meaning to their students may in itself help teachers take a step back, look at what they are teaching, consider their own reasons for teaching it, make judgments regarding the utility of various subjects and consider the best ways to go about teaching that subject matter that has withstood their critical attention.

Feuerstein discusses the ethical question regarding the mediation of meaning - what right does the mediator have to force his or her own personal meanings on the learner?

> The imposition of the meaning of the adult mediator on the child, or even adult, mediatee raises an ethical question: Who gives one the right to mediate, i.e. to impose one's own meaning and significance on the mediatee, determining by this not only his immediate behavior but also his future decisions?
>
> (Feuerstein & Feuerstein, 1991, p. 25)

This question has been one that has concerned many educators, and led to attempts at taking a more "objective" stance in their educational work in the hope that would allow their students to develop their own personal stands on issues that arise (Elliott, 1991). Feuerstein argues, however, that lack of mediation of meaning will result in the learner's inability to do just that. The role model that the mediator provides when he or she takes a stand on a particular issue, looking critically at situations, deriving meaning from them, coming to conclusions as a result of these considerations, and expressing these conclusions, is vital if the learner is to learn to do the same.

> Among parents and educators for whom the ethical basis of the mediation of meaning is called into question, this tendency [to refrain from mediating meaning] has become an ideology according to which the conveying of meaning by the mediator with specific intentions is unwanted in that it is considered too close to dogmatism and indoctrination. The followers of this "ideology" ignore the damage caused by interactions devoid of the mediation of meaning. This lack of meaning influences both the mediator and the mediatee, regarding the amount of interaction, its character and its power, not only between the mediator and the mediatee but also between the organism and the environment.
>
> (Feuerstein, 1998, pp. 55-56, translated from the Hebrew)

The view that the learner must learn to find meaning through contact with role models who externalize this process is another aspect of the situational character of MLE. The reciprocal nature of mediation also suggests the importance of the learner's frame of mind in the interaction, thus fulfilling at least this aspect of constructivist-based teaching. In addition, however, from a constructivist point of view, it would seem that mediation of the mediator's meaning need not preclude the mediation of the learner's own meaning as well. In fact, it may be that the students' active involvement in the attribution of meaning is not something that can be postponed to a later date, but rather must be incorporated into the learning process itself. For this reason, the students' mediation of their own meaning was given a central place, alongside my own, in our lessons throughout the year.

Additional Parameters of MLE

The parameters of intentionality and reciprocity, transcendence & meaning are considered to be necessary criteria for the very existence of MLE - if an interaction is lacking in one of these it is not considered MLE. Klein (1985) holds that the fourth parameter, mediation of the feeling of competence, should be included in the list of necessary conditions. Particularly in the case of mathematics education, whether it be that of college students or schoolchildren, the perception that the learner has of her ability to be successful in mathematics is of prime importance. And specifically for mathematics teachers, the belief that the teacher is competent enough to ensure the success of all her students is a precondition to the professional attitude that is necessary to enable her to do so.

In addition to the first three criteria of MLE, then, Feuerstein lists nine additional parameters that in his view are frequently found in mediated learning experiences but are not prerequisites for its existence. They are:

Mediation of a feeling of competence

Mediation of regulation and control of behaviour

Mediation of sharing behaviour

Mediation of individuation and psychological differentiation

Mediation of goal seeking, goal setting, and goal achieving behaviour

Mediation of challenge: the search for novelty and complexity

Mediation of awareness of the human being as a changing entity

Mediation of the search for the optimistic alternative

Mediation of the feeling of belonging

I will further elaborate some of these parameters as they arise in the account of my teaching during the year.

CHAPTER THREE

METHODOLOGY AND METHODS

METHODOLOGICAL CONSIDERATIONS

Action Research

Elliott defines action research as 'the study of a social situation with a view to improving the quality of action within it" (1991, p. 69). Carr & Kemmis (1988) relate particularly to the context of education:

> Action research is a form of self-reflective enquiry undertaken by participants (teachers, students or principals, for example) in social (including educational) situations in order to improve the rationality and justice of (a) their own social or educational practices, (b) their understanding of these practices, and (c) the situations (and institutions) in which these practices are carried out.
>
> (Carr & Kemmis in McNiff, 1988, p. 2)

The enactment of an action research project was in many ways a natural continuation of the educational activity I had been engaged in during my career as a teacher. The values and educational principles developed during my formative years as a teacher were greatly influenced by the open education and curriculum-reform movement in Britain and North America in the 60's and 70's (Neill,1960; Holt, 1964; Kozol, 1967; Kohl, 1969; Nuffield, 1971; Stenhouse, 1975). In these, traditional transfer models of education, whereby knowledge held by teachers must be transferred to the minds of children, were radically changed. Elliott (1995) comments on these new approaches to learning and teaching in schools that are important both for teacher education and for action research as well:

> Learning is viewed as the active production rather than the passive reproduction of meaning. Its outcomes are no longer to be assessed in terms of the match between inputs and predetermined output criteria, but rather in terms of the intrinsic qualities of being they manifest. When learning is viewed as 'active production' then it becomes a manifestation of human powers, e.g. to synthesize disparate and complex information into coherent patterns, to look at situations from different points of view, to self-monitor personal bias and prejudice, etc. The development of understanding is construed as the extension of the students' natural powers in relation to the things which matter in life. The manifestation of such qualities can be described and judged but not standardized and measured.
>
> (Elliott, 1995, p. 10)

The view of learning as active production, the focus on the power of human beings to change their knowledge structures, and the necessity of changing assessment methods to suit these assumptions are three elements which Elliott finds that lead to a view of teaching which facilitates

> ...an indeterminate dialectical process between public structures of knowledge and individual subjectivities. Its focus is on the process rather than the product of learning. It is directed toward activating, engaging, challenging and stretching the natural powers of the human mind.
>
> (ibid.)

According to Elliott, in order to develop a teaching practice that will resemble this description to the greatest extent possible, it is not enough to study learning, but rather it is necessary to look at the extent to which teaching supports this view of learning as well.

> Such a pedagogy requires teachers to reflect in as well as on the classroom process quite independently from any assessment they make of the quality of the learning outcomes. Pedagogy is a reflective process. It is process rather than product data which forms the basis of evaluations of teaching.
>
> (ibid.)

Examination of the components of this view of learning and teaching shows that these premises may carry over into a changed view of research as well. If learning is a process then the study of learning must focus on its processes rather than its outcomes. These complex processes cannot be assessed using a research paradigm taken from the natural sciences. If the goal of the research is to aid the improvement of teaching practice, then the researcher must not only understand these processes, but also search for ways of teaching which will actively enhance them. Modes of action research were developed to answer these practical needs of the field.

Corey (1954) notes a number of important questions that are often posed regarding teachers' knowledge, such as how teachers identify learning problems, how they analyze them, how they ameliorate the situation and how they evaluate change. He looks at the way in which these questions are often responded to:

> These kinds of questions, or ones like them, have to be considered in all kinds of teaching at all levels. The answers to them are often based on hopes, casual impressions, and superficial observations. When this is so, teaching becomes a rule of thumb - largely guess work. We never really know. We learn how to do better only occasionally. We don't seem to improve.
>
> (Corey, 1954, p. 208)

This description accurately depicts my feelings regarding my own teaching before beginning my research. Corey suggests becoming more objective and scientific by carefully observing the behavior of the learner, developing teaching-learning situations which are in tune with their needs and their character, and testing the results of these endeavours by gathering "dependable and appropriate" evidence of the resulting change.

> When we seek answers to important questions in this way, we become researchers. We study what we do. We begin to know more and more rather than to guess or hope more and more. This kind of research is often called "action research". It is a method of problem-solving that practitioners - teachers, administrators, supervisor - can employ to improve the quality of their decisions and actions. It is research conducted on the firing line.
>
> (Corey, 1954, p. 209)

Elliott proposes a definition of action research which emphasizes this symbiotic relationship between research and practice:

> Action research might be defined as *'the study of a social situation with a view to improving the quality of action within it.'* It aims to feed practical judgement in concrete situations, and the validity of the 'theories' or hypotheses it generates depends on...their usefulness in helping people to act more intelligently and skilfully. In action-research theories are not validated independently and then applied to practice. They are validated through practice.
>
> (Elliott, 1991, p. 69).

In his view, this identity between the acts of teaching and research is the attribute that defines action research and distinguishes it from other kinds of teacher research.

> The term 'action research' indicated a clarification of the research paradigm involved, and the relationship between research and teaching. They were not conceived as two separate activities. Teaching was viewed as a form of educational research and the latter as a form of teaching. In other words the two activities were integrated conceptually into a reflective and reflexive practice.
>
> (ibid., p. 30)

This singularity of purpose of teaching and research in action research allowed my attention to focus on both, while the understanding of each was supported and taken forward by the other.

Frameworks of Action Research

The above view of research forms the basis for the teacher-as-researcher movement (Stenhouse, 1975) that has gained momentum over the years (Altrichter et al., 1993; Atkinson, 1994; Elliott, 1991; Rudduck, 1986), and which served as the basic framework on which I based my own action research.

> To the extent that there is a teacher research/action research 'movement', there are particular values that its advocates endorse. These include a concern to democratise research by making it more accessible and accountable to teachers, thereby challenging the traditional theory-practice division. There are also concerns to empower teachers as a professional group by helping them to become more educationally and politically aware: reflection on action is seen as a means to that end...
>
> (Rudduck, 1986, p. 2)

There was one way in which my research was different from that portrayed above, and that is in the view of action research as a collaborative enterprise. Although there are some (Winter, 1996) who allow that the collaboration in research can refer to that between the researcher and her or his participants, in most academic formulations of teacher action research collaboration is seen as that between the teachers and external university-based supervisors, or as that of a team of teacher researchers, most often working in the same institution (Cohen, Manion & Morrison, 2000; Carr & Kemmis, 1986; Elliott, 1991; Zuber-Skerritt, 1992). Collaboration of this kind was not possible in my own research. My supervisors, who themselves were not action researchers, were situated at a university in a distant country, and e-mail communication with them was far from fluid. Because the research was being done as a doctoral dissertation, it was an individual enterprise, not a collaborative effort. I therefore needed to make do with the input and feedback received from my students, and from occasional conversations with colleagues who were themselves occupied in writing their own doctoral dissertations.

This same difference existed with regard to an additional form of action research with which I otherwise identified, emancipatory action research advocated by Carr and Kemmis (1986). They distinguish between three types of action research: technical, practical and emancipatory, while holding that only the last is 'true' action research. The first, the 'technical view' of action research sees teaching as a craft, and the goal of research as improving the efficiency of educational practice.

> Teachers' knowledge is assumed to be about the means available and their relative effectiveness under different circumstances.
>
> (Carr & Kemmis, 1986, p. 35)

In technical research the educational setting is taken for granted - there is no critical evaluation of the purposes of education, the effects of unjust societal practices, or the need to develop different kinds of skills, knowledge and attitudes in children.

The 'practical view' of action research sees education as a complex and fluid activity which is not amenable to means-ends technical control. Education is seen as a process which can be influenced, not controlled, through the work of expert teachers whose understanding is based on practical deliberation, and whose classroom intervention is both informed and .prudent.

> (The practitioner) uses professional judgment responsively, guided by criteria based on experience and learning which distinguish education processes from non-educational processes and which separate good from indifferent or bad practice.
>
> (Ibid. p. 37)

The third, 'strategic view', the one that characterizes emancipatory action research, sees education as an enterprise that is historical, social and political: it takes place in a historical context while holding before it a view of the kind of future the researcher hopes to build; it is a social activity which has social consequences over and above the effects it may have an any particular individual; and it is a political undertaking in that it determines the life chances and expectations of future citizens. A basic attribute of emancipatory action research is its view of the importance of the dialectical relationship between individual and group responsibility in educational settings: in order for the individual to effect change in his or her narrow teaching context, institutional social and educational relationships must be taken into account.

Having grown up in the sixties, and having learned the lesson of the burgeoning open education movement at the time, I had naturally acquired a critical attitude towards society, education and my own teaching, and therefore identified strongly with the view of emancipatory action research. I too felt the necessity of reexamining and reconsidering all elements of the educational process: its purposes, the kind of society it supports, the kind of knowledge it encourages. However, the centrality of collaboration in this formulation of emancipatory action research prevented me from describing my own research as such.

> The form of action research which best embodies the values of a critical educational science is emancipatory action research. In emancipatory action research, the practitioner group takes joint responsibility for the development of practice, understandings and situations, and sees these as

> socially-constructed in the interactive processes of educational life. It does not treat teacher responsibility for classroom interaction as an individual matter, but, on the contrary, takes the view that the character of classroom interaction is also a matter for school determination and decision-making.
>
> (Carr & Kemmis, 1986, p. 203)

Another major reason for my inability to view my research as emancipatory, quite likely connected with the above, was the fact that I was a relative newcomer to the field of higher education: when I began my research I had been teaching at the college for only three years. This situation caused me to feel what I consider to be appropriately modest; as an individual conducting action research, I certainly did not see it as within my power to

> ...create conditions under which the critical community can be galvanized into action in support of educational values...
>
> (ibid., p. 5)

While I was interested in improving my practice, and hopeful that my work with students would, eventually, have some influence on the way mathematics is taught in the country, I was not in a position at that time to, any direct way, transform my social context.

It would be possible, then, to characterize my research as practical action research in accordance with the categories described by Carr & Kemmis. Although for them this form of action research is insufficient in that it is impossible to isolate one's individual teaching situation from the wider context, Zuber-Skerritt (1992) suggests that all three forms can be seen as legitimate when seen as stages in the development of an individual's research.

> For Carr & Kemmis only emancipatory action research is true action research. In my view the three types are developmental stages, and it is quite legitimate to start with technical enquiry and progressively develop through practical and emancipatory action research. However, the ultimate aim should be to improve practice in a systematic way and, if warranted, to suggest and make changes to the environment, context or conditions in which that practice takes place, and which impede desirable improvement and effective future development.
>
> (Zuber-Skerritt, 1992, p. 11)

Having now worked as a teacher educator for more than ten years, and begun to discern the educational influence I have had on the understandings and teaching

practices of many of my former students, I now have the perspective and the confidence to see that my research has contributed to the wider practice of mathematics education in the city of Jerusalem. My standing in the college is now based on years of experience and (relative) success with my students, and I now feel that I am able to influence the institutional setting within which I work. I feel, therefore, that in accordance with Zuber-Skerritt, my practical action research has been a stage in my development as a researcher, and that I now have the ability to conduct my next action research project in the critical spirit of emancipatory action research.

Self-Study

A third framework of action research which has strongly influenced my own work is that of self-study teacher action research. Hamilton & Pinnegar define self-study as inquiry that takes advantage of professional autobiography, the wider context in which it is situated, and traditional academic theory.

> What is self-study? Self-study is the study of one's self, one's actions, one's ideas, as well as the 'not self'. It is autobiographical, historical, cultural and political and it draws on one's life, but it is more than that. Self-study also involves a thoughtful look at texts read, experiences had, people known, and ideas considered. These are investigated for their connections with and relationships to practice as a teacher educator.
>
> (Hamilton & Pinnegar, 1998, p. 236)

Self-study is the natural extension of Schon's formulation of the necessity of developing a tradition of inquiry into the epistemology of practice which takes into account the practical experience and reflection-in-action of professionals. According to Schon,

> ...when we reject the traditional view of professional knowledge, recognizing that practitioners may *become* reflective researchers in situations of uncertainty, instability, uniqueness and conflict, we have recast the relationship between research and practice. For on this perspective, research is an activity of practitioners.
>
> (Schon, 1983, p. 308)

The nature of inquiry into one's own teaching demands a model of research that allows the researcher to reflect on her or his own practice, while clearly referring to the self in the process. The model of social science research in which the researcher refrains from using the pronoun "I" cannot be suitable for this kind of inquiry.

I as a Living Contradiction

Whitehead suggests two major questions in which the "I" plays the central role, that are the focus of educational research: "How do I improve this process of education here?" and "How do I live my values more fully?" (Whitehead, 1993, p. 7). In my own case these questions may be phrased thus: How do I improve my teaching of this didactics course in early childhood mathematics? and How do I live my values more fully in doing so?

The issue of values is relevant to my work in two important ways. First relates to the extent to which I succeed in teaching according to my beliefs - in putting into practice those principles that I regularly preach to my students. The discomfort that I felt with my inability to put into practice much of what I viewed to be important in my teaching was a major source of motivation in my decision to carry out action research. One of Whitehead's major contributions is his view of the practitioner, who, in her or his attempt to live up to personally held educational values, simultaneously encounters their contradiction. Tiller uses the concepts of the "expressed" and "actual" curriculum to look at this contradiction:

> When we examine the relationship between the expressed and actual curriculum we will probably find that there are significant differences between the values we express and those that characterise our practical work....In this encounter with the self many people express a feeling of guilt. The distance between what actually takes place in the classroom and what is propounded as good teaching has proved a fruitful staring point for discussion and analysis of one's own everyday work.
>
> (Tiller, 1995, p. 49)

The tension between theory and practice is teachers' personal daily fare. Whitehead therefore suggests that practitioners consider themselves, the "I", as a "living contradiction" in attempts to improve practice by more closely aligning their values and their practice.

> [Whitehead] is keen to keep the teacher-practitioner at the centre of the enquiry. Unless we keep the living 'I' in our educational discussions, he maintains, action research loses touch with reality and becomes an academic exercise. ...the 'I' of each individual is his unassailable and inalienable integrity...the 'I' is a living, pro-active entity. It is vital that we acknowledge the force of the individual consciousness in interpersonal relationships. It is this force that makes possible, and its acknowledgment that encourages, a one-to-one relationship between persons that is fundamental to human enquiry.
>
> (McNiff, 1988, p. 37).

The question of my interpersonal relationships with my students points to another way in which the issue of values arose in my work. In this my experience of myself as a living contradiction related to the degree to which I felt I should or should not force my own values on my students. My desire to increase my educative influence with my students felt at times to be in conflict with two important values that I held: my liberal desire to refrain from forcing my own my values and beliefs on my students, and my understanding that their beliefs, in order for them to be truly be their own and therefore salient in their future work, needed to be personally and meaningfully developed by each student on their own. A potential solution lay in learning to teach in accordance with those values myself: to provide active experiences for my students that would better allow them to construct their own understandings and subsequent beliefs. Becoming aware of myself as a living contradiction, and seeing this as a catalyst to a process of improving my teaching, enabled me to begin a reflective journey which has resulted in a more personally acceptable mode of interacting with my students.

Living Educational Theory

According to Whitehead, action research, while being practical, is also the way to develop educational theory. Here, however, theory and practice are considered in an integrated fashion, each affecting and being affected by the other.

> Linguistic definition is inadequate as a medium for the clear expression of educational values. (It) needs to be supplemented by ostensive definition for a clear communication of the nature of educational values. The necessity of 'pointing to practice' (ostensive definition) prevents the separation of theory from practice in the constitution of educational theory.
>
> (Whitehead, 1993, pp. 36-37)

He addresses educational theory from the point of view of the practitioner by holding that the importance of theory for the practitioner lies in its potential to provide guidelines and guidance for a developing professional view of education.

> One of the distinguishing features of a profession is a body of theory which can help to justify and improve its practices. I hold the view that educational theory is a form of dialogue which has profound implications for the future of humanity because of the values it holds and because it is embodied in our practical lives in our workplaces and in wider society.
>
> (Whitehead, 1993, p. 6)

The resultant theory is what Whitehead calls 'living educational theory'. It is the individual's description and explanation of his or her educational development which,

along with the living educational theories of other practitioners, constitute a body of educational theory which is relevant to and supportive of the practice of teachers and others working in the field of education. In order for educational theory to be relevant for practitioners, it must have the power to explain their professional development.

> One of the major problems which has led to the discrediting of traditional forms of educational theory was that they could not produce adequate explanations for the educational development of individuals. A theory should also be able to answer questions concerning why things happen. In the approach to educational theory advocated here the 'why' questions are answered in terms of 'value'. Like Ilyenkov (1982) I take 'value' to be a human goal for the sake of which we struggle to give our lives their particular form.
>
> (Whitehead, 1993, p. 54)

Thus, although the concept of self-study may seem inward looking, it has the potential to contribute to more general, although practical, theories of learning and teaching.

> Teacher educators, many of whom were classroom teachers prior to entering the academy as university-based educators, engage in self-study both for the purposes of their own personal-professional development and for broader purposes of enhanced understanding of teacher education practices, processes, programs, and contexts...The purposes are not mutually exclusive. The former purpose typically has a largely practical (often pedagogical) focus and is usually self-oriented in that the general aim relates to the ongoing improvement of one's own (pedagogical) practice. The latter purpose has a broader aim more generally related to the production and advancement of knowledge about teacher education practices and the programs and contexts within which they are situated. Both purposes have to do with refining, reforming and rearticulating teacher education.
>
> (Cole & Knowles, 1998, p. 225)

This new form of educational theory has the potential to enhance the professionalism of teachers. While in other professions, research is closely bound up with professional practice, this has not been the case in the field of education:

> A tiny proportion of educational research- that is, funded research, carried out by proper procedures and then made public knowledge through publication - is undertaken by practising teachers: the vast majority of such research is conducted by university-based academics involved in teacher education who do not teach in schools.
>
> (Hargreaves, 1996, p. 3)

Hargreaves holds that educational research is, in contrast to that found in the field of medicine, non-cumulative, resulting in the lack of an agreed knowledge-base for teachers.

A few small-scale investigations of an issue which are never followed up

> inevitably produce inconclusive and contestable finding of little practical relevance. Replications, which are more necessary in the social than the natural sciences because of the importance of contextual and cultural variations, are astonishingly rare.
>
> (Hargreaves, 1996, p. 2)

The contribution of the living educational theories of individual practitioners to educational theory relevant for practising teachers can begin to ameliorate the situation as depicted by Hargreaves.

Ethical Considerations

This section seeks to discuss ethical issues relevant to my action research. While some of the problems met with in other forms of qualitative research, such as the attempt not to influence the course of the research by use of deception (not informing participants of the goals of the research, keeping one's true identity hidden...), may be seen to be irrelevant for self-study action research, others may be seen to change in their character or in their degree of acuteness, becoming either more or less problematic in this context. While self-study action research provides a unique opportunity to both teach and carry out research in which the research can directly affect and improve one's teaching, there are areas in which the demands of one would seem to conflict with those of the other. I will first relate to those areas of conflict between the occupations of teacher and researcher, and show how they may work themselves out to some extent, or continue to pose difficulties when the teaching and the research are carried out by the same person.

The primary subject of interest of theoretical research is the unalterable, the constant, the laws and features of nature, while that of teaching is the alterable: human beings who learn and develop and therefore change (Carr & Kemmis in Wong, 1995). Self-study action research assumes the alterable nature of the human being, including the teacher-researcher who is looking to develop and change, and makes no attempt to look at static situations (if those can indeed exist in the social world). Because the teacher researcher looks at change as it occurs in both his or herself and in students, any ethical conflicts that may result from attempts to 'capture' and 'hold' students' knowledge at any particular stage - by refraining from teaching the students for the sake of the research - would cease to exist.

Carr and Kemmis highlight an additional difference between teaching and research: the primary outcome of theoretical research is the development of theory, while that of

teaching is specific interactive classroom practice that is effective and justifiable. Self-study action research incorporates the theoretical and practical, resulting in the development of living educational theories of practitioners : theories that are personal and alive, and specifically connected to the changing context of practice. This idea of practical theory goes a long way to reducing any tensions that might arise between theory and practice.

Finally, according to Carr and Kemmis, the primary activity and goal of theoretical research is careful observation, reflection and inquiry, resulting in knowledge and understanding, while that of teaching is choosing the best thing to do in any particular situation and doing it. Self-study action research combines these activities and goals: it is our responsibility as teachers to choose our actions, to act in the best way possible, as well as to observe and reflect, thus improving action by increasing understanding. The difference that continues to exist lies in the different time-frames of the two activities: teaching considerations generally centre around immediate responses to occurrences in the classroom, while action research is a protracted attempt to solve vexing long-term problems of teaching. Because in self-study the researcher is the teacher, the short-term goal of doing one's best in interactive teaching continues to be of primary concern, along with the teacher-researcher's attempt to solve long-term problems.

Atkinson (1994) discusses two additional areas in which research and teaching may be seen as resulting in ethical conflicts in the degree of commitment to one or the other. The first, the question of time, is related to the differences in the primary activity and goals of teaching and research mentioned above. The problem of time and pressure was one of the major stumbling blocks during the action stage of my research. I needed to find the time to write and reflect while at the same time fulfilling my regular duties as a teacher. I needed to plan and think ahead so that I could carry out the research in a deliberate and considered way. While during this stage of the research I became increasingly impressed by the potential that systematic reflection on my practice had for pointing to promising solutions to vexing problems, I was, nonetheless over-burdened by the demands of data collection, the re-planning of my curriculum in terms of the theories that I was looking to put into practice, and the difficulties I experienced putting my chosen strategies into practice. I often felt overwhelmed by these demands. On the macro level, although I succeeded in functioning on a weekly basis, writing in my journal and planning the following week's lessons in accordance with an analysis of the previous week's, I often found myself buried so deep in the present that I was unable

to view the wider picture. On the micro level, although my lesson plans reflected my growing understanding, often plans made regarding specific actions or subjects to bring up in the lessons were forgotten in the "swampy lowlands" (Schon, 1983) of my actual teaching. Any reflection that I succeeded in doing during my interactive teaching was fragmentary - my actions during teaching could be characterized more closely as being intuitive rather than reflective. It is interesting, however, that this situation, while often extremely frustrating, did not seem to have affected the amount or the quality of my learning during this period, or my satisfaction with it.

A second area mentioned by Atkinson which is relevant to my own situation relates to the scope of the research:

> Research is sustained concentration on a very few things. These are studied in great detail and in theoretical terms. Teaching is action on a huge number of things, most of them fairly unconnected with each other.
>
> (Atkinson,. 1994, p. 387)

One of the difficulties that I came to identify in the plan of my research project was its all-encompassing scope. I was interested in improving my practice, and in order to do that I felt I needed to look at an entire range of problems, and to take into account multiple theoretical perspectives. While improvements in my teaching led me to feel that this broad perspective was useful and ethically in line with my responsibilities as a teacher, these advantages were likely attained at the expense of greater focus, rigorousness and systematic nature of research.

Atkinson also points to the time demands involved in writing up the research. Although this stage did not begin until after I had finished teaching this particular course, the demands it put on me did affect my teaching for a number of years afterward. I often felt that the extent to which my reflective journal had contributed to my teaching during the early stages of the research, was almost the extent to which the demands of writing it up prevented me from reflecting in any consistent way on my teaching in those subsequent years. Although in general it was clear that my teaching, and therefore my new students as well, benefited greatly from the learning that took place as a result of my research, problems which I identified in my teaching at this time had to be put on a back burner. Although the term "neglect" may be too strong to use here, I did have an uncomfortable feeling regarding the extent to which I was fulfilling the role of a responsible teacher as I had begun to understand it from my experience with my action research.

The problematic of researching my own students

In classroom research, where the respondents are also one's students, and have little or no choice as to whether or not they will participate in the project, the question of consent is particularly problematic.

One potential problem of researching one's own students, that of needing to assign grades for the course, was a problem that had concerned me from a pedagogical point of view since I had begun to teach at the college. In the past I had received permission from the head of the college to give pass/fail grades. However, because most students who took part in this year's course saw it, due to its largely practical nature, as one they could do well in, they preferred to receive number grades that would help to raise their grade average. The problem was partially solved through discussion with the students, resulting in the decision to allow them to assign their own grades as long as those were, in accordance with specific criteria, deemed reasonable both by them and by myself.

The question of informed consent in social science research is one that is dealt with differently in different situations. Cohen, Manion & Morrison hold that while the moral necessity of obtaining informed consent arises from participants' right to freedom and self-determination, when the situation does not threaten the participants in any way, the need for such consent is reduced.

> The principle of informed consent should not...be made an absolute requirement of all social science research. Although usually desirable, it is not absolutely necessary to studies where no danger or risk is involved. The more serious the risk to research participants, the greater becomes the obligation to obtain informed consent.
>
> (Frankfort-Nachmias & Nachmias, 1992 in Cohen et al, 2000)

Although in my own situation, there was no certain danger involved, my dual role as teacher and researcher could potentially give rise to conflict between my responsibilities towards my students and my desire to conduct valid and reliable research. The students who would be participating were first year full-time pre-service students for whom the course was a compulsory part of their programme. I was to conduct the research with only two of the three sections of the course, but at that point could not know which of these would be involved. The decision regarding which classes would take part depended on the schedule of each. Once the timetables of the classes were determined, because of the tight scheduling that characterizes the college, students in participating

classes did not in fact have the possibility of opting out.

I met with my potential student-participants at the end of the school year previous to the start of my research. I attempted to inform them regarding its goals (improving my practice as a teacher) and the methods that would be used in carrying it out (questionnaires, audiotapes of the lessons). However, because this was my first time carrying out action research, my explanation was surely far from complete.

I also took this opportunity to gather initial information regarding the students' attitudes to mathematics and mathematics education. This was done using a semi-structured questionnaire (see Appendix B). In order to relieve any worry they might have regarding the use of the data, I suggested that those who preferred to answer the questionnaire anonymously were free to do so. I continued this practice during the course of the year as well, both in regard to questionnaires and to any further feedback offered or were asked to write. Although a price was payed when I attempted to look at changes in students' beliefs over time, this was clearly necessary both from an ethical point of view, and in order to assure the students' consent to participate in the research.

The classes which eventually were included in the research were made up, in addition to these full-time students, also of in-service teachers. Although these teachers were not the focus of my research, their participation in the class made them participants in the study as well. During the first lesson of the year I once again described the research, while asking this time for the consent of all the students to begin audiotaping the lessons.

At that time I met with no opposition, and proceeded with my plans to audiotape the following week. Two weeks later, however, when I explained my idea of having a different student each week act as class reporter, one student stated that she refused to take part in the research. She was joined by two or three additional students, who voiced their opposition to acting as "guinea pigs" in my research. Again, because this was classroom research and the students were unable for technical reasons to transfer to the unresearched class, they had no choice but to be present in the research situation. Negotiations ensued in which it was decided that students who were opposed in principle to participating were freed of the responsibility of acting as class reporter, and I promised not to include data in my dissertation relating to any student that objected. During the course of the year the opposition waned, and other than that one vocal student who continued to object, the others eventually consented to take part.

Here I was faced with a situation in which the students' opposition caused me great consternation and difficulty, both practically in that the discussion and negotiation that ensued took up a significant amount of time and energy from the class, and emotionally in that I was unprepared for the virulence of the students' response. As a result I needed to find ways to deal with my own negative feelings towards these students and to continue to treat them fairly and be concerned with their welfare in spite of the difficulties and unpleasantness. This I succeeded in doing as well as possible with the guidance of the head of the programme, but the situation nonetheless highlighted for me the conflict that can arise from an attempt to act as both teacher and researcher, and caused me to take the students' participation less for granted for the duration of the research, and to inform them as fully as possible regarding the conduct of the research in its subsequent stages. This leads to a discussion of one of the ways in which the conduct of the research could be seen as contributing in one important additional positive way to the particular students who took part in its conduct.

The fact that I was conducting action research into my own teaching provided an important example of reflective teaching for my students. Although on the whole this could be seen as a positive effect of the research, it also caused some difficulty along the way. In Israeli society people are often afraid to show weakness, lack of understanding or doubt about what they do. Although a self-critical stance towards my work was natural for me, and I had at times included my students in my ruminations about my teaching in the past, the increased awareness that resulted from my action research resulted in quite frequent references to difficulties and uncertainties that arose in my teaching. These thoughts were in part the way in which I mediated for my students the importance of reflective practice and the potential it had for allowing teachers to learn and improve their practice. Nonetheless, this openness at times caused me difficulty, knowing that my honesty in regard to my work was not a culturally-accepted stance.

The Form of My Research

The Action Research Cycle

A number of different researchers have suggested models of action research which incorporate the aspects of both research and action. The research is generally seen as a cyclical process in which a problem is identified and studied, and then possible solution strategies are conceived, implemented, observed and evaluated. In this process a new problem is then identified and studied, with appropriate steps again being put into action in order to resolve the new situation, and so on...

Lewin (1941), who is seen by many as being the originator of action research, delineated four stages: (1) planning, in which a problem is identified and a strategy formulated for addressing it, (2) action, in which the first strategy is implemented, (3) observation, in which reconnaissance is undertaken to assess the effects of that step, and (4) reflection in which the evidence is taken into account and subsequent steps planned in light of the findings. These steps, although not necessarily in the same order, form the basic structure of action research now commonly referred to as the action research cycle. The cycle has subsequently been modified by different action researchers such as Kemmis, Ebbutt (see McNiff, 1988, pp. 26-32) Elliott (1991); Altrichter et al. (1993); and Lomax (1995). Some writers have expressed caution in the way these different versions of the action research cycle should be viewed. Rudduck holds that the action research cycle,

> in the hands of some academics, [has] been elaborated into an intricate and ritualistic set of hoops for teachers to jump through.
>
> (Rudduck, 1986, p. 7) .

Somekh warns of the dangers to the new action researcher of interpreting these models too literally:

> The models are no more than graphical tools to help us to conceptualise the action research process and, used in this way, they are helpful.
>
> (Somekh, 1995, pp. 342-343)

The action research cycle as it manifested itself in my own work has certain characteristics in common with the models presented by Whitehead (1993), Elliott (1991), and Rudduck, (1986).

Whitehead's Action Reflection Cycle

The action reflection cycle as presented by Whitehead reflects the order and the essence of my own research:

> 1) I experience a problem because some of my educational values are negated.
> 2) I imagine a solution
> 3) I act in the direction of the imagined solution.
> 4) I evaluate the actions
> 5) I modify my actions in the light of my evaluations.
>
> (Whitehead, 1993, p. 38)

Unlike models of the cycle which see the first stage as the reconnaissance stage, entailing a systematic analysis of the existing situation (Elliott, 1991; Altrichter et al., 1993), Whitehead holds that along with the experience of the negation of the practitioner's values comes an early attempt to imagine a solution. In my own case the difficulty of teaching early childhood mathematics to adults, resulting in a lecture-style of teaching which went against my understanding of effective teaching practices, led me to my first envisioned solution, teaching pedagogy "constructivistically". The experience of seeing former students teaching mathematics in the manner typical of the majority of teachers in the country at the time, and my exposure to the work of Feuerstein, led me to surmise that the provision of mediated learning experiences had the potential to increase the effectiveness of my teaching. For similar reasons I felt it necessary to situate the learning of my students to the greatest extent possible.

Elliott's Action Research Cycle

There are two ways in which my own cycle is reminiscent of the revised version of Lewin's model presented by Elliott (fig. 1). One was that I did not initially envision a single solution to the problem, but rather three as mentioned above. Intuitively I felt that no one of them alone would be sufficient to achieve my goals. Elliott's model allows for an overall general plan at the outset of the research involving more than one envisioned action strategy , where one step is implemented then evaluated, followed by additional steps, which, if still deemed appropriate, are then put into practice.

The second important way in which Elliott's rendition approximates my own reality is in his acknowledgment of the fact that not all strategies are successfully implemented the first time round.

> Implementation of an action step is not always easy, and one should not proceed to evaluate the effects of an action until one has monitored the extent to which it has been implemented.
>
> (Elliott, 1991, p. 70)

None of my chosen strategies were single-step, "do it or don't" strategies. A major part of my research involved learning to use those strategies that I thought would be helpful to my teaching. I needed to work out ways in which constructivist theory could be operationalized in my teaching. In regard to MLE and situated learning theory, I needed to both develop my understanding of the approaches myself and at the same time find ways to implement them. The degree of my success in implementing them clearly needed to be taken into account when attempting to evaluate their effectiveness.

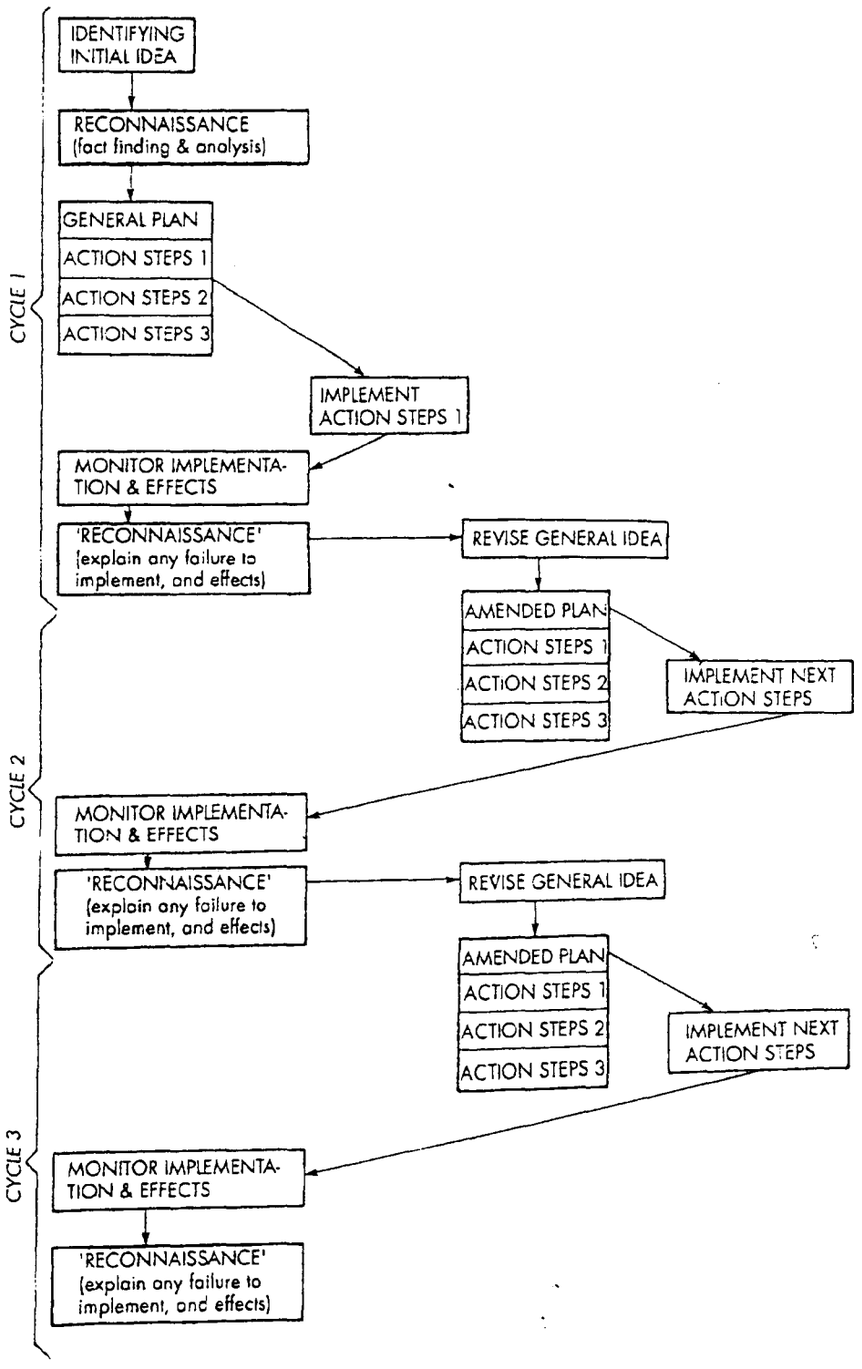

Figure 1 Elliott's revised version of Lewin's model of action research

McNiff's Non-Linear Cycle

McNiff addresses the problem of linear representations of the action research cycle in which one problem is dealt with at a time, evaluated, its ramifications taken into consideration, and a new problem is identified. She feels that this linear progression does not reflect the complexity of reality.

> Action research should offer the capacity to deal with a number of problems at the same time by allowing the spirals to develop spin-off spirals, just as in reality one problem will be symptomatic of many other problems.
>
> (McNiff, 1988, p. 44)

While in the interests of simplicity and focus it may be preferable to implement one action strategy at a time, my own reality did not permit such an approach. My own situation was one in which the complexity of my strategies prevented their full implementation as planned. In addition to this, in the course of my work additional problems cropped up at different points in time which demanded additional action strategies. This led to a complex picture of a number of spirals, of different lengths, each comprising the elements of problem identification, the envisioning of solutions, implementation of action step, and monitoring of the action step.

> Generative action research enables a teacher-researcher to address many different problems at one time without losing sight of the main issue.
>
> (McNiff, 1988, p. 45)

This complexity is expressed by McNiff's (1988, p. 45) portrayal of a three-dimensional series of spirals.

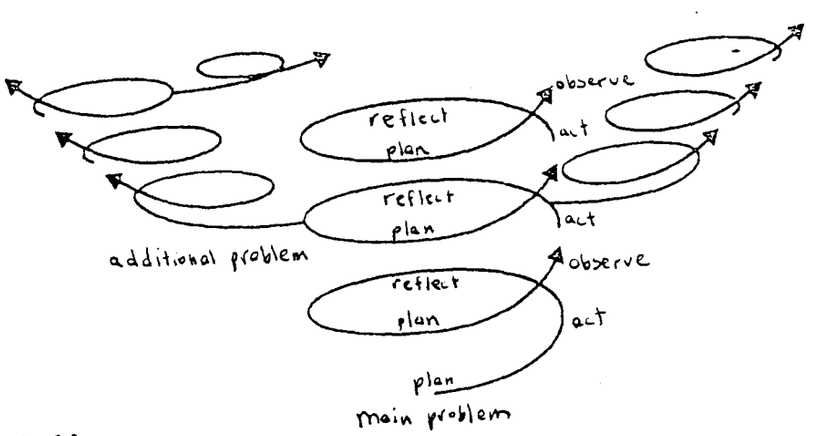

Figure 2 McNiff's diagramme of a three-dimensional spiral of spirals

VALIDITY IN SELF-STUDY

Realism and Nonrealism

Until recently the concept of validity in social science was seen in the context of positivist social science which assumed the existence of an external, objective world which a) can be known and b) can be known only in one objective, correct way. Instruments and procedures were created which would accurately and objectively measure the phenomena of this world. These were meant to counteract the inevitably subjective view of human scientists:

> The procedures of science are objective - not the scientists. Scientists, like all men and women, are opinionated, dogmatic, ideological...This the reason for insisting on procedural objectivity; to get the whole business outside of ourselves.
>
> (Kerlinger, in Smith, 1995, p. 134)

Validity as thus conceived is a concept that is at odds with a constructivist perspective towards learning and knowledge. If the objective world, assuming it exists, is unknowable to the individual, it becomes irrelevant to the development of the individual's knowledge. In the world of research, Smith (1995) refers to this constructivist position as nonrealism:

> If all observation requires an observer; and if an observer's observations are influenced by who one is...then it becomes impossible to know when what is outside of one leaves off and what is inside of one begins...This is precisely that question that nonrealists say cannot be answered, and as such, the realist assumption about an independently existing reality is fine - but of no consequence because it does not work.
>
> (Smith, 1995, p. 141)

In my own view, while human beings may never be able to arrive at an objective view of reality, it would seem that certain interpretations of reality may be closer to a "true" account than others. In research, data, while also affected by personal factors, can be relatively more "objective". The information picked up by audiotapes, for instance, while determined by the placement of the taperecorder, their quality of the tapes and the incomplete nature of audio data, nonetheless offer the opportunity to become aware of data that may otherwise remain hidden. The same may be said for other qualitative data collection techniques. By taking into account different sources of information as well as their contexts, the researcher can make an attempt to arrive at an understanding of the

situation which may be more in line with the nonetheless unknowable external reality which comprises the situation being studied. For example, the level of reading achieved by first graders and their motivation to read, while possibly affected by other factors as well, would seem to provide relatively objective evidence regarding the success of the teaching approach used. Phillips relates to this argument between different perspectives of constructivism:

> ...constructivists of both types - psychological and social - have to find room for the fact that our knowledge is about something. And whatever it is, it has to be granted a role in influencing our constructions...
>
> (Phillips, 1997, in Ball & Bass, 2000, p. 223)

Evers (1999) also argues in favour of what he calls a representationist view:

> If human behaviour were totally random, no language would be possible, nor interpretation, nor understanding. For creatures inhabiting an environment in which there are limited resources, there are advantages in forming trajectories which lead to the solution of problems at better than chance... performing a thousand...mundane physical acts is possible only against a background of sustained patterned, non-random, activity. The challenge to act on experience in a way that is more epistemically progressive than tossing a coin is the challenge to solve what we might call the navigation problem. It is the task of devising non-arbitrary strategies for meeting needs, achieving goals, solving problems, and, in general, getting around in the world while avoiding coming to grief sooner rather than later.
>
> (Evers, 1999, p. 271)

From this point of view it would seem that a practitioner studying his or her own practice may have difficulty arriving at a reasonably balanced account of that practice. The problematic of self-study is suggested by Carson (1997) in the implications he sees of psychoanalysis, particularly of Lacan's work, for the development of the teacher's identity and his or her desire to be seen as a particular kind of teacher. It is reminiscent of Whitehead's view of the practitioner experiencing the negation of his or her values in practice. Lacan sees desire as the lack that occurs as a result of the division of the self in which the "me", existing in the Imaginary order, "constitute(s) the subject with a (temporary) sense of wholeness" (quoted in Carson, 1997, p. 80) while the "I", the subject of language and culture, is constantly changing in relationships with others:

> The implications of Lacan for teacher education and reflective practice are twofold. First, the desire for a professional identity - to be seen as a teacher - can never be fully satisfied. The self desires wholeness, but because the self is divided, this lack can never be fulfilled once and for all...the ego itself has only the illusion of wholeness, one's self-image as a "person who teaches" is constantly being constituted and reconstituted in relationships with others (students, professional colleagues, parents). Second, much of what forms

> one's identity as a teacher occurs unconsciously...the existence of the unconscious has important implications for reflective practice because it suggests that objective access to the self and one's own practice will be highly circumscribed. At the very least, the role of the unconscious in identity formation points up the erroneous assumption in much of the reflective practice literature that we are able to stand over against ourselves and our practices as if we were knowing subjects standing in relation to an object.
>
> (Carson, 1997, pp. 80-81)

Whitehead's partial resolution to the problem is twofold: in the use of videotapes to record practice, and in his understanding of the contradiction of values as the motivating factor in practitioner research.

> When you view yourself on video you can see and experience...yourself as a living contradiction, holding educational values whilst at the same time negating them. Is it not such tension, caused by this contradiction, which moves us to imagine alternative ways of improving our situation? By integrating such contradictions in the presentation of our claims to know our educational practice we can construct descriptions and explanations for the educational development of individuals (King, 1987).
>
> (Whitehead, 1993, p. 71)

While my own circumstances did not permit the use of videotapes, the view of myself as a living contradiction became abundantly clear through audiotapes, through student feedback, and through my own reflections. My attempts to engage in triangulation (Somekh, 1983) techniques, while unable to provide an objective view of my practice, could enrich my understanding and bring it closer to an accurate account through offering me the opportunity to see it through multiple lenses. As will be discussed in the section on methods, the initial writing of my reflections based on direct impressions and memory of the lesson was a sketchy first attempt at understanding what took place. Listening to the audiotape exposed me to a different view, that picked up by a single tape recorder which recorded my own speech as well as that of the students who spoke clearly enough and/or were sitting closest to the tape recorder. Student reports of lesson segments allowed me to see it from a third point of view.

> ...triangulation is less a tactic than a mode of inquiry. By self-consciously setting out to collect and double-check findings, using multiples sources and modes of evidence, the researcher will build the triangulation process into ongoing data collection. It will be the way he or she got to the finding in the first place - by seeing or hearing multiple instances of it from different sources, using different methods, and by squaring the finding with others with which it should coincide.
>
> (Huberman & Miles, 1998, pp. 199-200)

Clarification of Values

While triangulation is meant to permit the researcher a view of the data that may be somewhat more "objective", in order for the reader to judge the significance of the account it is equally important that the researcher's values be explicitly stated. Whitehead's view of the practitioner accounting for him or herself as a living contradiction, reminiscent of Lacan's divided self, may be helpful here. Whitehead writes:

> In my own development I am conscious of attempting to overcome the experience of myself as a living contradiction in order to minimize the tensions between...values negated in practice and the current practice. I am also conscious of the need to give a form to my life and of the need for meaning and purpose...By drawing your attention to where the theory is being generated and tested in practice, I hope to emphasize that it is being generated and tested in practice and that it is embodied in the form of life of practitioners rather than it existing in a propositional form within textbooks on library shelves.
>
> (Whitehead, 1993, p. 56)

The desire for wholeness in our professional identity is the motivating force behind the enquiry. Its report fulfills three criteria of validity: the necessity to clarify a rationale for the research and show its educational significance as these are connected to value commitments; to articulate and justify intentions and beliefs; and to ensure the "constant dialectical interplay between researcher's values and actions where the values inform the actions and vice versa" (Stevenson, 1996, in Zeichner & Noffke, 199?, p. 8-9).

Controlling both the use of particular methods to assure a more objective stance, and the ways in which the practitioner can use the negation of his or her values to monitor the inquiry, is the practitioner's ability to take a critical stance. Notwithstanding Lacan's assertion that the individual cannot look objectively at his or herself and practices, a certain degree of this capacity would seem to be necessary for any researcher. Whitehead portrays this attitude as stemming from the researcher's desire to give a correct account:

> Because of a desire to give a correct account of the nature of educational theory I want to hold up the value-laden nature of my claim to knowledge for public criticism. I want you to understand and accept for good reasons the normative background of my ethical values.
>
> (Whitehead, 1993, p. 57)

The motivation for my own work stemmed from dissatisfaction with my teaching - in Whitehead's terms, from the experience of the negation of my values in my practice. As such, I have made clear in my account both the educational values that guide my practice and the difficulties that I encountered in putting these values into practice. The particular values which guided me have been clarified and developed through my review of the literature as well as through the presentation and analysis of the data. My self-critical stance, which engendered the project to begin with, was a basic characteristic of the research process and is hopefully apparent during its entire course.

Outcomes

Lather's (1991) discussion of validity is taken up from the point of view of feminist research. Although my research was not carried out expressly within an emancipatory paradigm, the fact that it dealt with mathematics and female students, and the disempowering situations which this combination often engenders, leaves little doubt as to the emancipatory potential of my work. Lather proposes the criterion of catalytic validity, which she defines as

> ...the degree to which the research process re-orients, focuses and energizes participants toward knowing reality in order to transform it"
>
> (Lather, 1991, p. 67)

The goal of much of the work that I did with my students was to change their view of mathematics as a subject thereby allowing them to feel both their own and their future pupils' potential in solving mathematical problems and dealing with mathematical situations. The transformation of my students into capable users of mathematics was, although important in itself, essential for the success of the project. The degree of attitude change among my students has been one of my main criteria in assessing the degree of success of the project.

An additional criterion suggested by Anderson (1994), outcome validity, is relevant here. This criterion looks at the extent to which a problem has been resolved as a result of the research, or the extent to which an action research cycle results in action. Stevenson (1996, in Zeichner & Noffke, 199?) suggests a similar criterion which points out the problem of "arrested action research" in which the research is aborted before action has been taken to either change the situation or resolve the problem.

It was not within the scope of my research to evaluate the lasting effects of the course on my students. My own teaching, however, both of the course and in general, has changed significantly. Evidence of change in my teaching was apparent during the course of the research, but its long-term effects were dramatic. The research has deepened my understanding of education and of myself as a teacher, and increased my teaching capabilities manyfold. Most significant for me was the increase in my self-confidence as a teacher. Although continuing to experience the negation of my values in my teaching, in Lacanian terms I now have a much greater sense of wholeness in my identity as a teacher . While my "I" continues to change in my relationships with others, I have developed the tools I need in order to direct and facilitate this change. These developments in my teaching will be discussed in the final chapter of this dissertation.

Generalizability

The question of the generalizability of research findings is an issue that must be seen differently in qualitative research than in large-scale quantitative studies. As opposed to quantitative research, which looks for generalization by searching for similarity of cause and effect among large populations, qualitative research looks at the uniqueness of particular cases. The information derived from individual cases can engender wider generalizations by either reinforcing findings from other studies, whether qualitative or quantitative, or by offering counter-examples which show these generalizations to be insufficient (Stake, 1995). However, there is an additional way in which particular cases can contribute to wider understanding, and that is through the resonance that the study may generate in the mind of the reader.

> Can readers accept subjective description? Often, the researcher's aim is not veridical representation so much as stimulation of further reflection, optimizing readers' opportunity to learn. Deborah Turnbull (Stake & Turnbull, 1982) called experiential learning "naturalistic generalization" and proposed that the qualitative researcher could organize the study to maximize the opportunity for naturalistic generalization. The study then could rely on actor and reader experience in ways already common in disciplined study of many kinds, such as historiography, philosophy, literature, and music.
>
> (Stake, 1995, pp. 42-43)

This concern with resonance is expressed by Newman (1999) as well. If, from a nonrealist stance establishing truth is not a concern, then what is important in the research for the reader is to see the problematic of his or her own work with new eyes.

> If the account lets me in, allows me to live in that fictional world and I can see connections to my world, it's likely to permit me to see my own work in new ways. I'm not looking for correspondence - I know my work situation is unique, as was the research situation being described - rather, I'm looking for what I have come to call "resonance" - does the account seem believable, does it help me think about the problematic of my working situation, does it help me name or reframe tensions in my own work so that I might do something about them?
>
> (Newman, 1999)

The study of my own teaching has the potential to combine with that of other teacher researchers to become part of the collective knowledge of the teaching profession, enlightening understanding in ways that are relevant to the lives of other practitioners. Most directly, my own experiences and understanding derived from my research may be relevant for other mathematics teacher educators. In addition, my concern with approaches to the teaching of subject matter that does not naturally lend itself to parallel experiential experiences on the part of teachers as it does to children, which developed as a result of the nature of early childhood mathematics subject matter, may make this research relevant to teacher educators working in domains that are similar in this respect. These may include language teaching, and possibly the teaching of history or geography. More generally, the use of a constructivist approach toward the teaching of pedagogy, and the examination of ways in which Mediated Learning Theory can be used to augment the learning experience, makes this research relevant to teacher educators in additional subject areas as well.

METHODS

Research Questions

The major question that I addressed in this project was: How can I improve my teaching of a pre-service mathematics didactics course in order to increase my educative influence on my students, thereby improving the likelihood that what they learn in the course will carry over into their future work as teachers of mathematics? This question is modeled on Whitehead's more general questions "How do I improve this process of education here?" and "How do I improve my practice?" (Whitehead, 1999). The way in which these questions are worded reflects the practical and dynamic nature of self-study action research as well as the centrality of context and the individual practitioner researcher in its conduct.

This question, however, was arrived at only during the course of my research, when I became acquainted with Whitehead's work. Until that time the questions which directed my research, and continued to play a major role in its conduct, were:

Research Questions

1. How can a pre-service training programme promote in students an attitude of responsibility for their own learning, and a feeling for the necessity for taking responsibility for the mathematics programmes they will teach.

1A)Focus on student teacher

(1Ai) How do student teachers perceive their role as teachers of mathematics for young children?
(1Aii) Is student teachers' independence and responsibility regarding their own learning process a precursor to their becoming independent and responsible teachers?

1B)Focus on the teacher researcher strategies

(1Bi)What are the teaching, i.e. scaffolding, strategies I can use to help student teachers:
(1Bia) take responsibility for their own learning
(1Bib) become responsible, autonomous teachers
(1Bic)promote in the students a questioning and research attitude toward their own teaching?
(1Bid) encourage them to see teaching as a dynamic problem-solving activity which necessitates constant reflection in order to evaluate past practice and to plan for the future?
(1Bie) encourage them to incorporate ideas presented by the teacher researcher or found in existing curriculum materials or elsewhere into the students' own curriculum framework in an active and thoughtful way?

(1Bif) encourage cooperative group work in order to reduce dependence on the teacher researcher and encourage their assuming greater responsibility for their own learning?

2. Assuming that previous experience as students in mathematics classrooms has provided student teachers with a narrow view of the possibilities inherent in mathematics teaching, what are the different kinds of knowledge, skills and abilities, and attitudes that student teachers need in order to become competent beginning teachers with a well-informed view of mathematics education?

2A)Focus on student teacher

(2Ai) To what extent are the necessary knowledge and attitudes existent in individual students at the beginning of the project and how do they develop over time?
(2Aii) What are the characteristics of the students' backgrounds, particularly of previous experience with mathematics, that influence their attitudes and their learning in this course?
(2Aiii) How do these characteristics influence their readiness to take responsibility for the make-up of mathematics learning in the classroom?

2B)Focus on the teacher researcher strategies

(2Bi) How can data from the above questions inform my practice and so improve the quality of my students' learning and teaching?

(2Bii) What are the kinds of mathematics activities that I should use with my students to help them:
(2Biia) get a feel for what mathematics is?
(2Biib) understand what it means to be engaged in mathematical activity?
(2Biic) develop a repertoire of mathematics curriculum activities that will allow them to feel capable of taking responsibility for their mathematics programmes?

(2Biii) How can I use the particular characteristics of the various frameworks in which I communicate with my students (coursework, lesson planning meetings, feedback sessions) to the best possible advantage?

3. How can I evaluate the effects of this course on the student teachers' practice?

3A)Focus on student teachers

(3Ai) To what extent do the discussions and reflection carried out in the lessons in the college affect their actual practice in the classroom?
(3Aii) Does the fact that more responsibility and independence are demanded of the students result in their offering and demanding more responsibility and independence from their pupils?

4. What is my role as teacher researcher?

(4A) How can I expose my uncertainty to my students without feeling a loss of face?
(4B) What kind of a relationship will develop between me and my students as a result of this open searching for solutions?

(4C) Will I succeed in acting as a role-model for this kind of attitude toward teaching?
(4D) What kind of negotiations must be carried out to ensure their cooperation and lack of resentment?

From these questions it may be seen that the research has a double focus: myself as a teacher, and my students as learners of a new profession. Each is intricately connected with the other. The first two major questions are subdivided into questions focusing on my students and those focusing on myself as teacher-researcher. The third major question focuses on the students, and the fourth on myself.

The questions in sections 1 and 3 reflected my belief in the necessity of teachers taking responsibility for their work. Nonetheless, some of these questions were found to be either unanswerable or unsuitable for my research. Although the questions in section 1B directed me towards teaching strategies that were important and relevant for my attempts to improve my practice, evaluation of the degree to which my teaching did actually promote responsibility in my students as future teachers (question 3Ai) was outside the scope of my research. Questions 1Aii and 3Aii, which looked at the connection between the students' degree of responsibility as students and the degree of responsibility that they would take as teachers were also found to be beyond the scope of this particular research project, and would seem to be unsuitable for self-study action research in general.

Question 2, which looked at students' backgrounds and their attitudes towards mathematics and mathematics education were both relevant and important for the progress of the research. Questions raised in section 2A regarding the students' backgrounds were incorporated into the pre-project questionnaire (Appendix B), and the first questionnaire of the year (Appendix C), and these issues were particularly addressed in a lesson in which the students discussed the importance of mathematics in small groups during one of the first lessons of the year (see Chapter 4). Questions in section 2B focussed on my choice of teaching strategies specifically regarding mathematics and mathematics education, and were central to the context-specific nature of my research.

Questions in section 4, which dealt with my role as teacher and researcher, were crucial to the research. Although questions 4A and 4B were not actually answerable in the form in which they appeared, they anticipated central issues, which were better addressed in the form of question 4C. This question, which relates to the extent to which I succeeded

in acting as a role model for my students in my attitude towards teaching, was answered to a great extent through the data obtained from the students' end-of-year feedback (Appendix E) and from their final assignments.

In hindsight I am aware, when looking at question 4D, that at the time this question was asked in a rather naive manner, under the assumption that negotiations must be carried out in principle, but anticipating no particular difficulty in the students' reception of the research. The reason for this may have been my feeling that the research was looking mainly at my own practice, and as such, would not engender any opposition. However, the difficulties that I experienced in the initial stages of my research highlighted the importance of this question, and the necessity of taking this issue into much greater account in self-study action research than I did at the time.

There was one category of questions which I eventually found to be missing: those concerning students' beliefs regarding mathematics and its teaching. Although this category may be understood to some extent in questions relating to previous perceptions regarding mathematics, and in the willingness of these future teachers to take responsibility for their teaching, and although the strategies I used almost always were those that encouraged change in the students' belief systems, it was only in the final analysis stage that I realized that this was possibly the major goal of my work with students. Including questions regarding beliefs may have helped to provide better focus to my work.

The Framework of the Research

The research was carried out with two classes of students enrolled in an introductory course called Didactics of Early Childhood Mathematics. The course consisted of weekly hour-and-a-half long classes. The part of the course studied, that which dealt with the development of number concepts, extended over a period of approximately six and a half months and included the first three modules of the didactics course. The first module, lasting from 28/10-19/11, included three lessons with each group. The second module, from 25/11- 7/1, included five lessons with the first class, held on Tuesdays, and six with the Wednesday class. The third module, from 13/1 - 13/5, included ten lessons with the Tuesday class and nine with the group that met on Wednesday. The course description can be found in Appendix A.

Both of the classes included pre-service and in-service teachers, 53 students in all. Four

of these students were new immigrants who were taking the course in order to receive their Israeli teaching certification. None of these were full-time students at the college. In regard to the in-service teachers, the education ministry at the time was in the midst of a campaign to ensure that all teachers in the country had, in addition to their teaching certification, a minimum of a bachelors degree either in education or in some other field. This encouragement on the part of the education ministry led to a situation in which, for a number of years, the education courses at the college were attended by a relatively large number of experienced teachers returning to complete their undergraduate education. The class given on Tuesdays had eight of these in-service teachers, approximately 33% of the class, while the Wednesday class had 4, approximately 17% of the class. Data collected on the in-service students and the part-time new immigrants was considered only to the extent that it influenced the overall interaction of the group, and questionnaires and assignments that were analyzed were exclusively those of the pre-service students. There were 37 pre-service students, three of which I did not consider - two of these because they were on maternity leave for extended periods of time during the year, and the third because of her opposition to taking part in the research. The research, therefore, was focused on 34 pre-service students.

The two classes studied were chosen because both of these were the last lessons that I taught on two separate days, enabling me to write my reflective journal while the lessons were still fresh in my mind. Because the research did not attempt to compare the two classes, but rather looked at my teaching and the educative influence that I had on my students' development, I looked at differences between them only when these seemed relevant to the purposes of the research.

The major methods that were used in collecting data were qualitative in character: semi-structured questionnaires, of the lessons, my reflective journal, students' reports of the lessons, students' assignments and occasional conversations with both colleagues and students.

Questionnaires

Three different kinds of questionnaires were distributed to my students, each with a different purpose. In all of the questionnaires the students were given the option of answering anonymously if they chose.

A. A pre-study questionnaire (see Appendix B) lasting 15 minutes was distributed to prospective students at the end of the school year prior to the beginning of the study. It included Likert rating scales (Cohen et al., 2000, p. 253) which were followed by open-ended questions, both of which were meant to provide me with an initial acquaintance with my students, as well as a baseline of attitudes and perceptions towards mathematics and mathematics education which could be used for comparison with data collected during the course of the research. This latter goal was necessarily somewhat compromised by the option I gave them of remaining anonymous.

B. At the beginning of the academic year at the college (approximately two months after they had started their fieldwork one day a week in the schools), a short questionnaire (15 minutes) was distributed in which questions were asked regarding the students' field placements - where they were placed, what opportunities they had to take part in mathematics lessons, whether these lessons were as they might have expected before they started, whether they felt the children are learning very much in the lessons...(see Appendix C). Although the questionnaire included ranking scales as well, the majority of questions posed were open-ended, in that my goal was to receive information about the specific idiosyncratic conditions in each student's field placement.

C. Two questionnaires (Appendices D & E) were distributed, one at the end of the first semester which roughly coincided with the end of the second module, and the other at the end of year, with the aim of receiving feedback from the students regarding their learning to date, the extent to which they had put into practice the ideas that had been introduced in the course, the extent to which they were satisfied or otherwise with the course, and, at the end of the year, regarding the educational principles they held at the close of the course. Both of these questionnaires included only open-ended questions. Comparison of these questionnaires with the pre-project questionnaire provided me information regarding the development of the students' thinking during the course of the research. This information was most useful for those students who signed their names on all of the questionnaires, although through analysis of all the questionnaires, trends could nevertheless be detected which included students who responded anonymously as well.

Audiotaping and Transcription

At the beginning of the year I began by audiotaping at least a half hour of each lesson with the two classes that were part of the project. The parts of the lesson that were taped

were chosen according to their potential to provide the greatest amount of relevant data for the research questions asked. In whole-class work I chose to tape portions that were planned to be more interactive and less lecture-like. In group work I taped the work of one group each lesson. The audiotapes were very useful in jogging my memory, and for the analysis of certain classroom occurrences. However, because I used only one tape recorder, in whole-class discussions the tape often succeeded in picking up only my own utterances, those of the students who were sitting close to the tape-recorder, and those who spoke loudly and clearly enough to be picked up by the microphone. The noise-level of the class during group work was often so high that the tape of the chosen group was often inaudible. This group- work was also recorded by the class recorder, however (see below), so the data was not lost completely.

In general I listened to the tape recording of the lesson only after I had begun to write my reflections on the lesson. This gave me the opportunity to reflect directly on my experience of the lesson first, and then to consider the additional data provided by the tape.

During the first part of the year I taped excerpts from both classes that I gave, but did not transcribe them immediately. Later on it was decided with my supervisor that, because this proved to be an overly heavy load, it would be enough to look at only the Wednesday class. The tapes of this class, however, would be transcribed in full at the end of each lesson. This new structure proved to be a heavier load still, mainly because of the hours it took to transcribe the material. In addition to that, I made the decision to continue to reflect on my lessons with the Tuesday class as well, because reflecting on that class contributed much to my teaching the following day. In the end I made the decision to include both classes in the research, taping, transcribing and writing my reflective journal on the Wednesday group, while simply writing the journal after the lesson with the Tuesday class.

Audiotapes from the beginning of the year that had not been transcribed earlier were transcribed, mostly in full, at the beginning of the analysis stage of the research. Long excerpts in the form of pure lecture which did not seem to add very much in terms of the research questions were not transcribed. There were also sections of the tape that were so unintelligible that they needed to be omitted.

The Lesson Reporter

In an attempt to provide an additional point of view, as well as to receive feedback on the lessons, a different student each lesson was asked to keep notes on the lesson. They did this for the whole-class parts of the lessons that were taped (up to 30 minutes), when they were asked to focus mainly on the interactions between myself and the students, and for the work of the group that was audiotaped as well, when they were asked to focus on the interactions between the students.

The request to have the reporters focus their reports of the whole-class part of the lesson on the interactions between myself and the students needed to be reiterated and spelled out time and again. At the beginning of the year, after discussing with them the focus of my research and the kinds of information I was interested in obtaining from their reports, I prepared written instructions which were meant to both jog their memories and specify what I hoped for in their reports (see Appendix F). These were first discussed with the whole class, and subsequently given to the students at the beginning of the lesson they would report on. In spite of these efforts, many reported on the lesson as they would if they were simply taking notes on the content of a lecture. Because the reports were done by a different student each week, they did not have the opportunity to improve their reporting through feedback. A number of times, when a student's report was done particularly well, I read parts of it out loud to the class so they would have a better understanding of what I meant. My recurrent efforts, however, often did not bear fruit. In hindsight, the instruction to look at the interactions between myself and the students appears to have been too limiting in that student exchanges in whole-class discussions, which could have provided valuable information regarding the influence of students' utterances on each other, were not often reported. In sum, some of these reports provided valuable additional information which provided me with the opportunity to see the lesson in a different light, some were only reports of the content of the lessons, while others, possibly the most "objective" ones, were strikingly similar to the tape-recordings and were useful when the tape failed to work or when the recordings were not clear enough.

In addition to the on-the-spot reports the students were also asked to write a summary in which they would reflect on the lesson and give me feedback. This summary was to be handed in the following week. Although these were often extremely helpful in the later analysis of the lessons, in terms of my reflective journal they were problematic in that I received them only after the journal entry for that lesson had been written. Although my

thoughts on these reports were sometimes incorporated in the following week's journal, for reasons of time and pressure this did not always take place. With time, again because of the pressures of the combined teaching and research situation, I neglected to remind them to hand in the summaries and from that point on only the more conscientious students continued to write them up.

My Reflective Journal

Altrichter et al. (1993) discuss the way in which the research diary "ensures that the data collection is not artificially separated from reflection and analysis" (p. 12). The diary contains data collected in the process of the teacher researcher's participatory observation in the field, reflections on the data, on one's role as a researcher and on research methods, and ideas and insights that can lead to the development of theoretical constructs which eventually may be used to interpret the data.

In the process of my research I kept two separate research diaries. One was a hand-written research diary that accompanied me through the entire period of my research, before beginning the fieldwork and continuing after its termination. In this diary I noted my thoughts and insights connected with the research. These were most often stimulated by my reading of the literature and I learned to always keep the notebook at my side as I read.

The other was a reflective journal written at the computer during the fieldwork stage in which I noted and reflected on data from my lessons. Generally I began writing the journal before listening to the tape of the lesson, and then added to it after having listened to the tape. This provided me with the opportunity to reflect directly on the lesson the way I originally experienced it, and then to reflect again on the way the lesson showed itself on the tape. This was one of the ways in which I attempted to triangulate in my data collection. There were instances, however, when circumstances did not allow me to write either reflective notes or to write in my journal immediately after the lesson, and then I felt that I needed the tape to jog my memory in order to begin writing.

In my journal I both described and analyzed the events of the lesson and reflected on possible ways to deal with any problems that became apparent. I focused both on problematic aspects of the lessons which were causing me to feel dissatisfied with my handling of different situations, and reflected on possible strategies which could help me

handle them more effectively. I also looked at successful aspects of my teaching, trying to understand and isolate the reasons for my success in order to be able to utilize this understanding in the future. The reflective journal provided me with a running commentary of the developments during the year, allowing me to review and compare occurrences, thereby bringing out themes and patterns in my thinking and in my behaviour.

There were times when, due to technical difficulties, or because I had forgotten to turn on the tape, I was forced to write my journal based solely on my direct impressions of the lesson as I had experienced it in real time. This occurrence was somewhat eased by the account of the lesson I received from the lesson reporter, but the lack of the recording was nonetheless keenly felt.

Student Assignments

There were two types of assignments which served the function of providing me with important information regarding the students' understandings and development. One of these was the summaries written by group reporters of assignments given for small-group work. These provided both information on the students' understanding of the subject matter under consideration as well as feedback on the activities and the way in which I introduced them and took advantage of them.

The other was students' individual assignments. At the beginning of the year I had not yet realized how important a source of data these could be, and only recorded my own reactions to the assignments. Towards the middle of the year their value became apparent, and I began to type into my student database quotes from their work as well as my reactions. At the end of the end of the year many of the students agreed to have me keep their assignments to refer to later on in the formal data analysis stage. The students' assignments provided me with information regarding their understanding of the main issues discussed in our lessons, the extent to which they took these ideas into account in their work with children, and the extent to which they were successful in putting them into practice.

Opportunistic Data

Rich sources of both ideas and data were occasional conversations with both students and colleagues which occurred during the process of the research. Problems

encountered were often given new light, and new ways of looking at ordinary occurrences were suggested. While these conversations were not actually reported, they were often incorporated into my reflections in my journal writing.

DATA ANALYSIS

The Focus of my Analysis

Bauersfeld (1988) discusses analysis from an interactionist perspective:

> As a product from social interaction, the patterns develop from the mutual reflexive expectations and interpretations of the actors and their related moves, from the implicit "obligations for action", which are typical for the institutionalized educational processes, and from the teacher's and student's routines as acquired across many shared classroom experiences. It is the network of relations among these constituents that produces the pattern of interaction...Teacher and student(s) constitute the reality of the classroom interactively.
>
> (Bauersfeld 1988 p. 37)

In order to answer the question of how to increase my educative influence on my students I studied interactions that took place during the course of the year, the reflexive relationships between myself and the students, and between the students themselves, within the developing classroom community. There are different perspectives from which these interactions may be viewed. Bauersfeld illustrates how his perspective shifted during the course of his research and the effects this has on the analysis:

> For the active researcher, each position arrived at relativizes the preceding perspective. This can lead him to disregard previous perspectives, and even to a relative blindness caused by the deliberate shift of focus...And there is no easy integration as conclusion...
>
> (Bauersfeld, 1988, pp. 30-31)

This difficulty in integrating perspectives is, for Bauersfeld, a positive state of affairs in that

> ...only competitive descriptions and contrasting issues have the power to produce challenge and to disquiet and break the customs of self-evident routines and explanations. There is no other way to distance oneself from habit and to allow for effective critical comparisons and reflections.
>
> (ibid., p. 42)

Although not a deliberate analytical strategy, my own perspective shifted during the course of my research. During the initial period the focus of both my data collection and analysis was the social interactions that took place in our lessons. This focus was reflected in the nature of my reflective journal as well as in my instructions to the lesson

reporters directing them to focus on interactions both in the whole-class segment of the lesson and in the group work. At this early stage I was concerned with the development of the pedagogical content knowledge of my students. One of the major reasons for the importance of ensuring positive interactions between myself and the students was the necessity of introducing them to an alternative view of mathematics and mathematics pedagogy which was at odds with many of their perceptions and beliefs. At this point I needed to be concerned mainly with the educative influence on the students that I, as their teacher, could effect. My social constructivist theoretical framework, my attempt to provide mediated learning experiences for my students, and the use of MLE theory as an analytical tool directed and facilitated my focus on these interactions.

Towards the middle of the year my perspective shifted as I began to discern the generation of a classroom culture in which particular pedagogical norms were beginning to come into being. While I had begun to see progress in my students' understandings earlier in the year, this seemed to have occurred as a result of my own efforts to provide appropriate educational experiences. From this point of the year on, however, development could be seen which I felt was not directly related to my own efforts, but to the influence of the interactions between the students themselves. These developments were in keeping with the social constructivist literature (Wheatley, G., 1991; Yackel & Cobb, 1996; Cobb & Yackel, 1998) , and focused my attention on the development of norms which had the potential to promote reflection and thus exert their own influence on the further development of both the classroom community and the individuals within it.

At the conclusion of the school year, during the formal analysis of the data, my perspective shifted once again. In considering the effectiveness of the course I focused my attention on the individual not as part of the classroom community, but as individual future teachers who would or would not attempt to operationalize their learning and insights in their future teaching. I then began to feel that the effects of the course needed to be examined by looking at the beliefs of the individual students, and examining the degree to which these were commensurate with the values and perspectives of mathematics education conceived according to social constructivist lines.

Coffey and Atkinson (1996) suggest that in qualitative analysis , the units of analysis of standard quantitative research can be replaced with "units of narrative". Each of these foci in my work led to the use of different units of narrative. In the first stage the

primary unit of narrative was my own teaching. During the second stage my major focus was on the developing community of practice. In the third stage my unit of narrative was primarily the perceptions and beliefs of individual students.

The Ongoing Analysis of Action Research

As suggested in the previous section, data analysis is an ongoing process in action research which begins during the data collection stage. The action research cycle of problem-posing, planning, implementation, and evaluation has important effects on the ongoing teaching process as well as on the research process.

> Collecting data will probably push you to attend to aspects of your teaching and your students' learning that you might otherwise overlook in the usual day-to-day work of teaching. Because it asks you to step back, make connections, and develop interpretations, analyzing data often leads to new perspectives on familiar things. So both steps are critically in this process of making familiar things seem new and strange. Together these processes can make you question what you are certain about, and lead you to greater certainty about some aspects of teaching and learning that you might have doubted.
>
> (Freeman, 1998, p. 90)

Altrichter et al (1993) refer to this ongoing analysis as the constructive stage of analysis. The first element of this stage is the "reading" of the data in which the data are scrutinized in order to recall the events of the situation under investigation. In my research this reading of the data involved my initial direct reflection on the lessons, listening to the tapes, and reading the student reporter's write-up of the lesson. The analysis of the data was done through the writing of my reflective journal.

The second element of this constructive stage is selecting data, in which

> Important factors are separated from unimportant ones; similar factors are grouped, complex details are sorted and (where possible) simplified.
>
> (Altrichter et al., 1993, p. 122)

Toward the beginning of the constructive stage of the research I found myself writing freely in my journal, with little regard to focusing on particular issues or organizing the data. It was only when particular themes began to emerge that I was able to begin to separate out the more important issues and to classify the data in accordance with these.

The third element is presenting the data, in which the data are reduced in order to facilitate its perusal. During this stage of my research this was done mainly through the

writing of periodic summary journals for which I reread the data collected until then, and reflected on the issues that I felt were most crucial, pressing, or problematic. One of the ways in which these periodic summaries were important was that they offered me the opportunity to take into account the student reporter's summaries of the lessons which I had received only week or two after the lesson itself. In this way, and as a result of my developing understanding, my reflections in these journals were more than summaries, taking into account additional data from these student summary reports and utilizing the perspective gained by coming back to the data at a later date.

The final element of this stage is interpreting the data and drawing conclusions:

> Relationships are explained and a practical theory or model constructed to fit the situation which has been researched. The theory or model should relate to the research focus.
>
> (Altrichter et al., 1993, p. 122)

Over the course of the year various themes emerged, different ones taking precedence during different periods. The original research questions, being somewhat too wide in scope, became continually more focused through the theories that were emerging from the data. A number of theories were developed in the course of the year, but in the final analysis not all of these were deemed helpful. Others were continually reinforced by the data from the field as well as by the theoretical literature, and developed into the central theories of the study.

MLE as an Analytical Tool

From early on in the year I began to use the parameters of Mediating Learning Experience as criteria in the analysis of my practice. In my journal writing I would often analyze interactions with my students in terms of the parameters that could be discerned. It may be seen that often my level of satisfaction with the lessons was directly connected with the evidence I could see of these criteria in my teaching. In addition to using them in these early stages of the analysis, I found them helpful in the later analysis of the data as well.

Writing as Analysis

> ...I consider writing as a method of enquiry, a way of finding out about yourself and your topic. Although we usually think about writing as a mode of "telling" about the social world, writing is not just a mopping -up activity at the end of a research project. Writing is also a way of "knowing",

> a method of discovery and analysis. By writing in different ways, we discover new aspects of our topic and our relationship to it. Form and content are inseparable.
>
> (Richardson, 1998, p. 345)

Writing is an integral part of the entire action research process. Richardson discusses poststructuralism as a form of postmodernism which is helpful in considering the writing involved in social science. From a poststructuralist perspective, language is not considered merely a reflection of reality, but also a means of producing meaning, thus creating social reality. Social constructivist theory would seem to reinforce this perspective, wherein the various forms of writing involved in the research process are seen as important contributors to understanding. During the constructive stage, the writing of my reflective journal allowed me to analyze, clarify and develop assumptions, understandings, hopes, and theories concerning both my teaching and my research.

This view of writing highlights an important aspect of self-study action research. The research process contributes to the development of the teacher researcher in many ways, and one of those is through writing.

> Language is not the result of one's individuality; rather, language constructs the individual's subjectivity in ways that are historically and locally specific.
>
> (Richardson, 1998, p. 349)

Thus the insights with which I was awarded through analyzing the data in this constructive stage of the analysis were an integral part of the analysis. They allowed me to either modify my earlier analyses of many problematic situations in my teaching, thereby improving my practice in at least some of these, or to identify and strengthen practices of which I had been unaware until then, that the analysis showed to be particularly appropriate and effective.

The Data Analysis Stage

> In actual practice, we read and reread a portion of the data and provide labels - usually notes in the margins - that identify a meaning unit. This process is called coding...We refer to the first broad categories as 'bins' into which the coded data can be given an initial rough sort.
>
> (Ely et al., 1997, p. 162)

At the beginning of the formal analysis of the data I decided to use my list of research

questions as these initial 'bins'. This served initially to focus my thinking on specified purposes of the research. Subsequently it helped to focus my analysis on the issues that were emerging as most substantial, while allowing me to put aside issues that had come to seem secondary or not directly relevant to the present work. The questions were organized according to major questions and subquestions. Therefore, along with developing codes for the issues that were emerging from the data, I numbered the research questions in such a way that I would be able to quickly find the specific question or questions which were addressed by any particular piece of data. The following is an excerpt of one journal entry which was coded in this way:

> Then I gave them the problems to classify for homework. I think this part of the lesson took just the right amount of time - although it is complicated to understand the first time, I think we did enough to prepare them to do the assignment successfully. And the assignment will help to fortify their understanding. This was definitely the right instance of when to give a short bit of homework. **eff t'g(effective teaching) - diff frmwrks (different frameworks) 1Bii c (question 1 part b, third part of second subquestion)**When I gave them the homework I was once again mediating intentionality and reciprocity by telling them how important it is to understand this point, I was also mediating the feeling of competence by telling them that although it is confusing at the beginning, if they take the necessary steps they will understand it quickly, and also mediating goal achieving behaviour. **eff t'g (effective teaching) - MLE crit (MLE criteria) 1Biv (question 1 part B, fourth subquestion), resp (respnsibility) 1Bia, (question 1 part B, first part of first subquestion) hmwrk (homework) 1Biic (Question 1 part B, third part of second subqestion)**

I then used the computer to paste the data segments in documents arranged according to the questions. This process allowed me to sift out questions which had been less relevant to my developing research focus, while obtaining a more detailed and encompassing picture of the more central questions.

After this initial organization of data, in which certain themes were beginning to emerge as foremost, I felt it necessary to re-examine the data, this time allowing myself greater freedom to discover issues that had not been foreseen by the research questions. Tesch provides a rule for developing an initial organizing scheme:

> Note the topic, not the *content*! When you look at a piece of data, ask yourself, 'What is this *about*? Don't pay any attention yet to what is said, i.e., to the substance of the statement; you will deal with this at a later stage. (pp. 142-3)
>
> (Tesch, 1990, in Ely et al., 1997, p. 169)

I began labelling the data according to topic, some examples being 'teaching problem',

'student participation', 'understanding', 'student thinking', 'problem-solving', 'mathematical content', 'why', 'spattering', 'MLE', 'role model', 'reflection-in-action', 'group work' 'listening', 'control', 'accepting', 'principles', 'attitudes', 'responsibility as learners', 'responsibility as teachers'... After labelling the data from a number of lessons in this way, when I had approximately 40 different labels, I sorted these into more general categories. These categories were: 'responsibility', 'MLE', 'them & me', 'principles, norms & beliefs' 'teaching problems - mine & theirs', 'teaching strategies', 'role and place of the individual', 'context' and 'methodological problems'. I then colour-coded my data using different-coloured markers for each category. These codes were then used for the further textual analysis of my reflective journals, the student reports, the periodic summaries I had prepared, and in the lesson transcriptions. During this process I looked for 'meaning units' (Ely et al., p. 162) which could contribute to my growing understanding of the themes that were emerging from the data. Although these were most often classified into existing categories, as I continued coding the data the categories were sometimes altered to keep them relevant and suited to the data. Both simultaneously and subsequently I began to look for relationships between the categories, the culmination of which was the writing of an outline which I assumed would provide the structure of the dissertation.

I then prepared a preliminary draft thesis in which I structured the dissertation according to the major themes that had emerged during my analysis. During the writing process, in my attempt to justify my arguments, I found myself more critically searching for data which would verify them. I looked at the different categories, reviewing the colour-coded data and looking for evidence which could reinforce or put into question my understanding of the themes that had emerged until then. This process at times led me to refute interpretations that had previously seemed obvious and whose refutation had escaped my previous analyses. Here, as discussed above, the writing process played a crucial role in the analysis, often resulting in new understandings and new ways of looking at the data, and at times causing me to change my original perceptions and understandings.

Because of my concern with demonstrating the way in which the action research process had unfolded in my research, I became dissatisfied with my original intent to structure the presentation of the data in my dissertation according to themes. To this end I prepared a time-ordered matrix (Miles & Huberman, 1994). This matrix was organized along one axis according to the three teaching modules which I was looking at, and along the other, according to the themes that I had been considering previously. It

highlighted the continuing developments in my teaching, including the difficulty I was having in putting into practice different strategies that I had decided to use, the growing participation and understanding of the students, and the differences in the character of each of these modules. This matrix provided the backbone which allowed me to rewrite the draft in a more linear fashion, presenting the active process through which I gradually learned to implement the action strategies I had determined to use at the outset of the project, as well as the problems and insights that had led me to devise additional strategies.

The subsequent write-up of my findings provided an additional opportunity to sift through the data, looking at both the particulars and the general meanings that were emerging from them. In describing the three consecutive modules of the didactics course I attempted to provide enough "thick description" (Geertz, 1973) for the reader to get a sense of the way the course unfolded from the point of view of both myself and the students, and of the interactions between us. To this end it was necessary to depict the content of the course as well, to provide a context within which to view our interactions, but also to understand the educational goals to which I aspired through those interactions.

Hitchcock & Hughes discuss the role of theory in qualitative research:

> Obviously, the qualitative researcher does not enter the field in an empty-headed fashion, indeed, school-based teacher-researchers have to have some guiding ideas or notions. However, theory in qualitative research is not prescriptive, it is rather creative and open-minded and therefore operates in a different manner.
>
> (Hitchcock & Hughes, 1995, p. 30)

This theory is derived from such sources as personal knowledge and experience, educational theory and reports of the work of other practitioners. Like Whitehead (1993) in his construct of living educational theory, Hitchcock & Hughes emphasize the importance of a reading of the literature in the development of this theory:

> ...despite the difficulties the teacher-researcher can and should be able to draw from a broader body of educational and social science knowledge in however a limited fashion in order to make sense of and hence move towards the analysis of the data collected.
>
> (ibid., p. 303)

Bauersfeld approaches a similar conclusion, emphasizing the inadequacy of existing theories of analysis:

> The attempts towards holistic interpretations [of numerous re-plays of videotapes] get into trouble in many aspects, for which no adequate theories of analysis have been developed so far. Inevitably, therefore, the interpretations of classroom scenes, and of teaching-learning problems in particular are cases not only of an application of scientific theories but also of a creative use of everyday understandings and shared (sub-) cultural knowledge.
>
> (Bauersfeld, 1988, p. 32)

There was a long period, while I was preparing to write the concluding chapter, in which a reinterpretation of the data became the main focus of my thinking. In addition to the eyes-on analysis of the data, I continuously mulled over the entire research process, trying to make sense of the research as a whole. This process had begun during the last part of the year in which I taught the course, and continued until the dissertation was ready to be submitted. I would imagine that it will continue in the future as well. It was in this attempt to make sense of the whole that there was a continuous shift in what I felt to be the underlying meaning of the research. This was not the analysis of particulars, but rather the fruit of my struggle to understand for myself, and to explicate for the reader, what was special about my work and in which ways it could contribute to the growing data base of research on teacher education.

CHAPTER FOUR

MY STUDENTS

Who my students were

The group of students which I looked at was made up of 34 full-time students of the regular early childhood four-year programme at David Yellin Teachers College in Jerusalem. Each year the programme includes at least one day a week of teaching practice guided by a supervisor who, in addition to observing them in their work and conducting one-on-one supervisory discussions every two or three weeks, also teaches them a course in pedagogy called "Analysis of the Teaching Process". The practicum and the accompanying didactics course together make up the central component of their programme. In the first year of the programme the students' teaching practicum takes place in pre-school classrooms, in their second year, the year in which I teach them, they work one day a week in either first grade (6-7 year olds) or second grade (7-8 year olds) classes. It is only in their third year that the students spend longer periods of time in the classroom, at the teaching level that they choose (pre-school, first or second grade). In their second and third years, although the college courses are given from October through the beginning of June, their practical work begins on September 1, with the beginning of the children's school year, and continues until the end of June. At the time of this study in their fourth year most of the students were already acting as full-time teachers and they returned to the college one day a week to complete their course work. At this time their work was accompanied by a supervisor at the college.

In addition to this central pedagogical course and courses in literature, literacy, Jewish studies and the arts, in their first year at the college the students took introductory courses in psychology, in which they studied a wide range of psychological issues, and educational philosophy in which they learned about educational concepts and ideologies and discussed the connection between theory and practice. Because issues in educational psychology made up only a small part of the psychology they learned, when they entered my course at the beginning of their second year most of the students had only limited and superficial knowledge of the cognitive development of young children. During their second year they took one additional compulsory course in psychology: The Family, the Pre-school and the Child, which dealt with the connections between these three elements and their effect on the child.

The Didactics of Early Childhood Mathematics was a required course. During their first year the students took a course called "The Basics of Mathematics" in which they were introduced to new, higher level mathematics content that was meant to deepen their understanding of mathematics as a discipline. This course did not consider pedagogical issues. At the time of my research project, the status of a third year course in mathematics pedagogy, which I had taught until then, was shaky. It had included observing the students teaching mathematics in their practicum. The subsequent decision to discontinue it meant that the students' mathematics teaching was observed only occasionally, and usually by supervisors who themselves often had little knowledge or understanding of advances in mathematics education.

As described in the previous chapter, both classes that were considered for this study were made up of a mixture of full-time pre-service students and of practising teachers who were taking the course for in-service credit. The data collected regarding the in-service students is reported only to the extent that it seemed to have a direct affect on the work of the pre-service students.

The Pre-project Questionnaire

Because it was not yet clear which two classes would actually take part in the following year's project, at the end of the school year prior to the beginning of the project, I distributed questionnaires to all the first year full-time pre-service students. The questionnaire was meant to gather information regarding their backgrounds and their attitudes towards mathematics (see Appendix B). Although there was no mention of my research intentions in the questionnaire itself, I explained to the students before they responded to them that it was being given to them as part of a research study that I would be carrying out with some of them during the following year. The students were offered the option of answering the questionnaire anonymously. Of the 34 students who were included in the research, 32 answered the questionnaire, 2 apparently being absent on the day that it was given. It is the responses of these 32 that are reported here, 16 of whom chose to remain anonymous.

From the point of view of the research, this questionnaire was given as part of the first action research cycle. The general purpose of the research had been determined by five years of previous experience teaching this course in which I had identified a number of key problems. It was now necessary to focus on the particular population with whom this research was to be carried out.

From the point of view of my teaching, starting with a questionnaire allowed me to take into consideration, to some extent at least, the perceptions and perceived needs of my students. In accordance with a situated view of learning, I felt it was important to find ways to connect my teaching with the previous experience and present reality within which my students found themselves at the college and in their student teaching placements.

The first section of the questionnaire contained four statements reflecting attitudes towards mathematics and mathematics education which the students were asked to rate on a scale from one to five, in which 1 represented "not at all" and 5 represented "to a great extent". The final statement was meant to evaluate the extent to which they had encountered conceptual work in mathematics in the kindergarten classes where they were placed during their first year at the college. The results were:

Table 1

	not at all	not very much	neutral	quite a bit	to a great extent	total
I like mathematics	0	5	11	6	10	32
I liked mathematics in elementary school	4	3	6	10	9	32
I liked mathematics in high school	5	5	7	9	6	32
I feel comfortable with the prospect of teaching math in school	0	5	10	10	7	32
In the preschool where I worked this year they worked on developing children's mathematical concepts	6	5	9	8	4	32

The most striking feature of their responses in this first part of the questionnaire is the relatively positive attitude towards mathematics that is expressed by the majority of students. Only five (15%) indicated that they do not like the subject, while sixteen (50%) said that they do. Regarding their elementary school experience with the subject, nineteen students (60%) said they liked it, and only seven (22%) said that they didn't. In high school the situation is somewhat different - ten students (31%) said they did not like mathematics during their high school years, but there was still a high proportion of those that did - fifteen students (47%). Regarding their degree of comfort with the thought that they would have to teach mathematics to children, sixteen students (50%)

indicated that they were perfectly comfortable with the prospect.

Regarding their experience in the pre-schools where they had been doing their student teaching until then, eleven students (34%) indicated that they had seen more than minimal work on mathematical concepts in their classes. This was important information in that it indicated that in the pre-school context in which they had worked during their first year many of the students had not had the opportunity to see the way mathematics might be taught at this age-level.

In the next section of the questionnaire I described a typical class of each of two imaginary teachers, and asked the respondents to note the positive and negative traits of each as they saw them. The descriptions were as follows:

> The first teacher, Nitza, likes mathematics and enjoys teaching it. She begins her lessons with explanation given to the whole class together. She then invites a few children to the blackboard to solve a problem while the others are watching, and asks the class whether they agree that the solution is correct. Finally she has the children work individually in their workbooks, each at their own speed.
>
> The second teacher, Gila, has the children sit in groups according to the subject that they choose to study on that day. There are various manipulatives put out at each table. While working on the tasks the children consult with each other. Towards the end of the lesson Gila asks the different groups to report on the work they have done.

An analysis of the responses to this question shows that sixteen students (50%) expressed a clear preference for the teaching style of one teacher over that of the other. Of these, fourteen (87.5%) chose Gila's more open teaching style, and only two (12.5%) chose Nitza's more traditional style. Eleven students (34%) were more neutral in the way they related to the two styles, pointing out what they saw as the positive points of each of the styles. It is interesting to note that seven of these more neutral students (64%) were those that had reported in the preceding section of the questionnaire that they liked the subject of mathematics. Assuming that an enjoyment of mathematics indicates previous positive experiences with the subject, it may be possible to interpret this data as indicating that the numerical ability of these students allowed them to succeed in and appreciate the more traditional type of mathematics teaching that they experienced as children. On the one hand, this finding potentially had ramifications for my teaching in that it was in keeping with my previous experience teaching the didactics course where I found that students who experienced success as children in mathematics classes at times found it difficult to accept more progressive views of the

way the subject might be taught. On the other hand, the fact that they had not chosen Nitza's conservative style of teaching as being preferable may have been an indication that they had begun to learn about and appreciate alternative ways of teaching in general and might be expected to be somewhat open to an alternative style of mathematics teaching as well.

A corresponding finding showed that of the six students who said they did not like mathematics at all, four of them (66%) chose Gila's teaching style as that of preference, with only two (33%) mentioning both positive and negative points of each. None of these students chose Nitza's style of teaching as being preferable. It was possible that for them, as I had found with other students in the past, an alternative view of mathematics teaching would be accepted as a welcome event. Of the nine students reporting a neutral attitude towards mathematics, five (55%) clearly chose Gila's teaching style, two (25%) mentioned positive and negative attributes of each, one (11%) chose Nitza's style, and one did not answer the question.

Comments which were repeated by more than one respondent regarding the positive points of Nitza's lesson were: all the children hear the explanations (3 responses); the children work individually in the workbooks (5 responses); she knows how to explain the material (4 responses); she likes teaching mathematics (3). The fact that these were the most frequent responses may indicate a number of perceptions regarding mathematics education: that by ensuring that all children hear explanations most will understand them as well; that the role of mathematics teachers is to explain; that having the children work in the workbooks at their own pace is an important way of individualizing the learning; and, clearly, an understanding of the importance that a teacher enjoy what s/he teaches. The three main negative points were: not all the children are at the same level and understand her explanations (9); calling the children up to the blackboard is threatening and puts pressure on the children (7); having the children judge the work of those who solved the problem on the board (3). These indicate the understanding that although many children may understand the teacher's explanations, there are always some that don't; and that the emotional factor must be taken into account when teaching mathematics.

Positive points regarding Gila's lesson were: she understands the children (3); the children discuss and work cooperatively in small groups (5); the children work with concrete materials (4); the children are free to choose (3). These responses may indicate the influence of previous courses taken at the college which encouraged a more

progressive approach to education than most of the students had experienced as children. Negative points were: the possibility that too much freedom might prevent children from learning everything that they should (4); technical difficulties of keeping order and being able to get to all the children (3). These last points show an awareness of the difficulty of practising the principles of a more open teaching approach.

The last question of the questionnaire related to the students' expectations of the course in pedagogy that I was to teach, and asked whether they had any requests. There was a wide variety of responses to this question. Those which were repeated a number of times were: to receive the tools they would need in order to teach mathematics efficiently so that the children would understand (6); that my way of teaching should be clear and simple (8); that they should learn how to explain the material properly (4); that the course should be interesting (4); and that the course should be relevant (3). Somewhat less progressive than the attitudes reflected by their responses to the previous question, these expectations reflected both a traditional view of mathematics education, in which it is the teacher's role to explain the material to the child, and a utilitarian view of the didactics course (Calderhead & Robson, 1991; Stuart & Thurlow, 2000), in which they need only learn the tools to teach the subject matter efficiently. Together the responses reflected a complex situation in which I would need to find the ways to both build on the beginnings of a more progressive view of education which was in the process of developing in many of the students, and the many traditional perceptions of mathematics and mathematics education which were held by many of the students (Ball, 1990b).

Beginning-of-year Questionnaire, November, 1997

The purpose of this questionnaire was to get an idea of what kinds of mathematics lessons the students were currently observing/participating in the schools, and a preliminary evaluation of the extent to which these mathematics lessons fit in with their prior perceptions and their expectations of these lessons. The questionnaire was filled out by 30 students, four students being absent from the lessons on the day it was given.

When asked to describe a typical mathematics lesson in the class that they were working in, three main types emerged from an analysis of their descriptions: Nine students (30%) reported a typical lesson as one in which the teacher begins with a whole-class explanation of the work to be done, followed by individual work in their workbooks; Five students (17%) described lessons in which the children work on their own during

the whole lesson, usually in the workbook but sometimes in their notebooks; Three students (10%) described lessons in which the children take an active part in games or other mathematical activities.

In the next question the students were asked to rate the extent to which the type of lesson they observed in their class was in accordance with their expectations.

The results are displayed in the following table:

Table 2

	not at all	not very much	no expectations	to some extent	very much	total
whole class teaching followed by individual work	0	0	4	5	2	11
independent work only	1	3	1	0	0	5
games / activites	0	0	0	0	3	3
whole class lesson	0	0	0	1	0	1
students' intentions unclear	0	0	0	3	4	7
did not see enough lessons to determine a typical lesson	0	0	3	0	0	3
total						30

The last question asked the students to describe the ways in which the lessons did or did not agree with their expectations. The responses of the twenty students that answered this question can be sorted into six categories:

1) Four students' comments related to the use of the workbooks (the text in this case). All four criticized the emphasis on the work in the workbooks.

2) Three students related to the teacher's teaching, all of these mentioning the clear explanations of the teachers.

3) Six students mentioned either the level of the material taught or the level of the children's mathematical knowledge. Of these, two students felt the level of the material

to be too low for the children, one felt the level was right for them, and three felt that the level was in accordance with their expectations, without relating to the extent to which it is appropriate for the children.

4) Three students related to the active nature of the work done in the class.

5) Two students related to the understanding of the material by the children, one feeling that the children did not understand what the lesson was about, and the other noting that she felt that : "the work with the rods is very technical and didactic. But in any case the children really understand the material".

6) Two students related to the use of concrete materials, one noting the fact that the teacher used them, and the other had "expected there to be change in the approach to learning to an approach that is based on more concrete work, but it doesn't look like it".

Analysis of these responses shows a high level of consensus regarding a structure of mathematics lessons in accordance with many students' expectations. Of the eleven students (55% of the 20 respondents) who reported that the typical lesson that they observed was of the form "whole class teaching followed by individual work", all indicated seeing these lessons either as in accordance with their expectations or at least not conflicting with their image of what mathematics lessons should look like. This is somewhat reinforced by the fact that of the five students who observed lessons where the children worked independently with the workbooks for the whole duration of the lesson, 4 (80%) indicated that this was not what they had expected to find. It is noteworthy that the three students who found themselves in classes where mathematics was taught using mathematical activities and games all reported that these were very much in accordance with their expectations. Was it just coincidence that those three students happened to have different expectations than many of the others, or was this a situation in which a student, having seen a more open and flexible lesson than those that were known to them previously, felt this to be a model of teaching which is in keeping with the more open view of general teaching that they were learning about at the college? This latter interpretation may better suit the findings of the previous questionnaire in which the majority of the students, while choosing Gila's more open teaching style, expressed expectations of the didactics course in keeping with a more traditional view of mathematics education. It would seem that when faced with an alternative to traditional mathematics teaching many of them at this point were able to see its advantages, but were not yet capable of imagining for themselves what these lessons might look like.

Further Information

Additional preliminary information about my students was obtained from a task which I gave them in one of the first lessons of the year, when I asked them, in small groups, to consider why they felt, if at all, that it was important for people to learn mathematics. Some of the responses that they listed to this question were: that it helps to understand ones' salary breakdown and one's social insurance payments, to figure out change in the grocery store, to figure out what a percent discount means in money-terms, to have a feeling of what amounts mean, to be able to estimate, so that people won't cheat you, and to develop analytical skills. The latter point, the development of analytical skills, was mentioned in only one of the ten groups which participated in the activity in the two classes.

The reasons they gave seem to indicate an understanding of the practical utility of mathematics in everyday life, but show little consideration of any wider educational value of learning mathematics. A number of explanations for this phenomenon may be put forward - first, that the question led them to think in practical everyday terms. Second, that most had never thought about the connection between mathematics and analytical thinking. And third, connected with the previous explanation, that they themselves, in their years of schooling as children, had never experienced the kind of mathematical activity which encourages thinking.

Summary

When looking at the overall data regarding the students' attitudes and experience with mathematics and mathematics education, what emerges is a group of students whose previous experience as children and present experience as pre-service student teachers, is for the most part of a traditional mode of mathematics education. On the other hand, there seems to be a beginning of a more progressive view of education which, when identified in mathematics teaching situations, is recognized and appreciated by many of the students.

CHAPTER FIVE

THE FIRST MODULE - SORTING AND CLASSIFYING AND CONCEPT DEVELOPMENT

For the first module of the year I chose to focus on the use of Mediated Learning Experience. There were two main considerations behind this decision. One was the function of this module as an introduction to the ideas and ways of working that would characterize this didactics course. The use of MLE, allowing me to point out the students' own current conceptions and beliefs about mathematics and mathematics education, would begin to indicate possible ways of connecting these with the alternative view of mathematics that my teaching represented. Second was my interest in having the criteria of MLE become a lens through which they would subsequently be able to view and analyze both their own teaching and mine. This goal was related to my growing understanding, from recent study of Feuerstein's theories, of the centrality of MLE in ensuring effective teaching/learning situations.

The subject matter of the first module was the subject of sorting and classifying. In addition to providing an introduction to the three essential criteria of Mediated Learning Experience, the lessons had two separate but intimately connected foci:

*They introduced the students to various sorting activities, from the sorting of assorted materials closely related to the everyday life of children to the sorting of more abstract, structured sets of geometric shapes.

* They dealt with the idea of concept-building on the part of the learner, and how this is done through the individual's sorting and classification of stimuli to which he or she is exposed.

This chapter presents the work done in this module and the ways in which my practice was based on my theoretical understanding of MLE and its connections to constructivism and situated cognition.

The Use of MLE

The importance of mediation as the initial focus of my work may be seen by looking at the three essential criteria of MLE: intentionality and reciprocity, meaning and

transcendence.

One of the major challenges of teacher education is to help students get past the belief that they already know what teaching is. Because of their long "apprenticeship of observation" (Lortie, 1975) which they experienced during their years as schoolchildren, many students feel that they are in teachers college to learn the techniques that they will need in order to become the kind of teachers that they already know.

Stuart & Thurlow describe their students as entering the methods class with "a utilitarian focus, expecting that they would learn motivating strategies and techniques for classroom practice" (Stuart and Thurlow, 2000, p. 114). Ball (1988) describes a similar situation specifically in terms of mathematics education:

> Students do not expect the course to challenge what they already know about teaching mathematics. They want to get better at what they know math teachers have to do: explain, show and tell.
>
> (Ball, 1988 , p. 12)

My students' beliefs, evident through their responses to the first two questionnaires, were to a great extent in agreement with these descriptions. As discussed in my treatment of the theory of Mediated Learning Experience, any one instance of MLE must include the first three criteria: intentionality and reciprocity, transcendence, and the mediation of meaning. When I needed to begin to counteract my students' taken-for-granted beliefs, intentionality was evident in my desire to introduce them to a new way of looking at mathematics education. In my efforts to ensure their reciprocity, I mediated both the fact that there were new and important things that they could learn from this course as well as their significance for their teaching and their pupils' learning. This mediation of meaning needed to be both general and specific - they needed to understand the meaning of the approach as a whole and that of the individual activities of which it is comprised.

In order to arrive at this understanding, the students needed to compare their previous experiences of learning and teaching and the taken-for-granted beliefs that these engendered, with their experiences with this novel approach to mathematics education. In turn, comparisons needed to be made between their experiences in the course and their implications for future practice. This transcendence, the bridging between different manifestations of the same idea, could allow them to arrive at a more

generalized understanding of the concepts and phenomena involved. The mediation would hopefully engender a willingness to deal with theoretical issues in addition to meeting their desire for practical tips to use in their teaching, and initiate a process whereby they would develop new understandings and modify their beliefs regarding the essence of mathematics and its teaching and learning. The action strategy that I chose to achieve these goals was to have my students begin to understand the importance of looking critically at mathematics education by constantly asking the question "why".

Asking the question "why"

I began by having them examine carefully the premises behind both the activities I had them take part in and the mathematics that is taught to children in school.

> If pre-service students do not bring their beliefs to a conscious level and articulate and examine them (Lasley, 1980), they will perpetuate current practices and the status quo will be maintained. This is unacceptable given that... many of the beliefs teachers and children hold are counterproductive to the teaching-learning process. As pre-service teachers begin their careers, they will be in a position to break this cycle, but they will be incapable of doing so as long as beliefs of which they are not cognizant drive their classroom practices.
>
> (Stuart & Thurlow, 2000, p. 119)

In my case this was done with a dual purpose in mind: to have them consider the importance of mediation, particularly the mediation of meaning, and to have them focus on their own thinking regarding mathematics education. The question "why" had the potential to allow them to critically examine their taken-for-granted beliefs about mathematics education. As they questioned their beliefs they could discover that not everything they had always taken for granted could stand up to scrutiny. There was also a very clear value message present in this strategy, that of the importance I saw in maintaining a critical attitude toward educational practice, and the importance of justifying educational decisions that could affect the lives of children.

When the students entered the class at the beginning of the first lesson I asked them to write their names and draw their faces, with any prominent attributes they might have, on stickers which they were to wear during the lesson. On the tables there were 'treasures" of random objects (sets of assorted small toys, parts of old games, puzzle pieces, buttons, screws, ribbons, old pencils...) which they were invited to play with freely.

I began by putting out various treasures and telling them, as they came in,

> that they're invited to play. After a few minutes I told them that usually when I tell them to do something there's a reason for it, and I'd like them to start thinking about why I do everything I do in these lessons.
>
> (Journal, 28/10/97, lines 5-7)

The statement that there is a reason behind my actions, although seemingly a truism, was a way of pointing out the fact that reasons are not usually questioned - that college teachers, as schoolteachers, often pronounce tasks to be done and are rarely called upon to justify their choices. Students are usually not expected to wonder about the reasons behind their actions.

As they were playing I began to move around the class, looking at the names written on the stickers and looking closely at their faces.

> While they continued to play (sorting, making an attribute train, building upward, feeling the materials) I began to read off the names and go around one by one, looking at them and their stickers, to try and remember their names.
>
> Then I asked them why I had done that with the stickers - they actually showed me ways in which I had mediated that I hadn't been aware of - that I had made it very personal, that I had broken the ice, that I wanted to get to know them - if I had taped it I could have given other reasons they mentioned. Nobody actually said to remember their names - that sounded quite prosaic next to the ideas they brought up.
>
> (Journal, 28/10/97, lines 11-18)

The statement that my own ideas sounded quite prosaic compared to theirs gives an indication of the enthusiasm with which I greeted their responses. This was one way in which I encouraged them to come up with their own ideas and to attribute their own meanings to the situations experienced in our classes.

During this first module there were many situations where I asked them to consider the reasons behind my decisions to do particular activities with them. During the first lesson I asked them two further 'why' questions: first why I had given them the opportunity to play freely with the materials, and then, in small group discussion, to consider why they felt, if at all, that it was important for people to learn mathematics.

In all of these instances I gave the students the opportunity to attribute their own meanings to the work we were doing. The example of finding reasons for learning mathematics provided valuable information regarding the students' beliefs and understandings regarding mathematics and mathematics education. Although there was

the occasional response that related to the intellectual benefits of doing mathematics, almost all related to the usefulness of knowing mathematics for everyday activities such as banking, shopping and looking after household finances.

The students were also asked to consider the reasons why I had introduced sorting as the first subject of the year. I had them discuss these ideas in groups and then, as the groups read off their lists, we discussed the various points.

Donna: Any way that you decide to sort is correct. In other words, you don't have to legitimize your participation, because there is no right and wrong.

Rachel: Nice. It's actually an activity without sanctions.

Donna: The development of thinking.

Miri: To learn regularity ('according to a law' in Hebrew)

Rachel: What do you mean?

Miri: Because I sort according to red, blue, green, according to the first letter, according to shape

Rachel: Maybe. Because you act according to certain rules.

Michaela: On the one hand the game is open, on the other there are rules which you have to keep.

Goni: There are all different kinds of rules. That's the thing. It's not one rule and that's it. It's also open.

...

Miri: And our second idea is that it is a social activity. Because one person can't sort alone.

Hadara: Yes he can.

Miri: He can, but then he wouldn't be playing the game we just learned.

Rachel: And this group?

Sorrel: You asked why you chose to start with sorting. I think it's the basis of life. We sort without even noticing it.

Rachel: When do we sort?

Sorrel: All the time.

Rachel: And from what age do we start?

Sorrel: From the age of zero.

Hadara: I think it's a basic tool which allows us to change in life.

Rachel: "A basic tool which allows us to change in life.' That's nice.

Sorrel: To recognize attributes...It enriches our knowledge.

(Transcript, 5/11/97, pp. 1-3)

The above segment reveals some beliefs that the students may be beginning to question. First, Donna's comment about not needing to be right in order to be able to participate in a math lesson implies the belief until now that mistakes are seen as a negative phenomenon, and that a person who makes mistakes has nothing to offer to a mathematical discussion. She also mentions there being no right or wrong - presumably because of the assumption that in mathematics there is always one correct answer and/or one correct way of solving a problem. Michaela's comment about the game being open, as well as Goni's that there are all different kinds of rules both seem to indicate their surprise that a mathematics activity can be flexible and open. This might shed light on a prior belief that mathematical activity must be highly structured and constrained by very clear rules that leave no room for maneuvering. Hadara's and Sorrel's comments at the end, that sorting is a basic tool which allows us to change in life, and that it enables us to enrich our knowledge, clearly demonstrates a beginning understanding of the centrality of classification in the cognitive development of the individual. It might also indicate an initial shift away from the purely utilitarian view of mathematics education that had been evident in the students' responses to the question of why they felt it was important that people learn mathematics. On the other hand it is possible that it indicates a prior understanding of the potential of mathematics as an intellectual pursuit which, through activities such as this one, might be realized.

A few of the students tried doing sorting activities with children during the course of the week, and, in the next lesson, I asked them to report on what they had done and how it had gone. Nora, an in-service kindergarten teacher, said that the children in her class sort all the time. I asked the students how they might be able to take advantage of the fact that they do so, a discussion which, without my intending it, revolved around the question of mediation.

Gail: All the time you can remind them, you can emphasize what they're doing. You can show them "you're taking all the pinks" or to ask them "which colour did you choose from all of the colours?". And if you put all the bags in this bin and in this one only paper - it tells them that all these are bags.

Rachel: One of the important abilities of teachers is the ability to express in words what the children do on their own. To make them aware of what is happening, to talk to them, to ask them questions that

will cause them to think...It's extremely important because when an adult - and that's what we're going to talk about in a few minutes - when an adult mediates for the child what is happening, the child can learn more and learn more meaningfully than when he or she just interacts with the physical environment.

Nora: Our mediating is just to tell them -

Rachel: Good - that's what I wanted to talk about... Is mediating only telling them things, or is it asking them questions as well? For example, if there were children that sorted something, I have two possibilities - either to verbalize what they were doing, in order to turn their attention to what they were doing, or to ask them "What did you do here? Why did you decide to put these here and those there? That way I can really come from where they are, so that the discussion is understood and meaningful. Here you learn from them, from the things they bring up, according to which they acted.

(Transcript, 5/11/97, pp. 5-6)

This exchange, while again looking at the centrality of classification in cognitive development, indicates Gail's understanding of the role of the teacher in realizing its potential. In my own rewording of what she said, I pointed out that making these comments is one way in which teachers can mediate for their students. I also took advantage of this opportunity to connect the use of MLE with the operationalization of constructivist theory - one can mediate by telling, or one can mediate by asking. Paradoxically perhaps, rather than having them work out their own meanings, here I mediated mine by expressing my clear preference for having the learner consider the meanings that s/he attributes to situations. What I neglected to do was to mediate the fact that the many instances in which I had already had them answer the question 'why' were a reflection of this preference in my own work with them.

Almost all the instances where I asked the students to consider the reasons behind the choice of what to teach and how to teach were based on occurrences that either happened in our lessons or in their own previous experience. Both of these are apparent in the following conversation:

Rachel: Another attribute of mediated learning is... the mediation of meaning. As you saw both in last week's lesson and in today's, we keep on working on the question of why - what are the reasons behind the things we do. Now why is it important to ask the question why?...I have reasons for doing what I do and I want the child to understand those reasons...Why is that so important? Why does the child have to know why?

> The truth is that often teachers don't say why they are doing things. They have the children do something, and they do it because that's what the teacher said to do.

Ilana: That's pretty bad.

Rachel: What's not good about it? Why is it important that children should know the reasons?

Hadara: So that they will understand, so they will think.

Miri: So they'll know that when you do things there is some logic behind them.

Tzila: ...even when the teacher doesn't explain, there are children who ask of their own accord.

Goni: It's a question of education usually. As soon as a child grows up in an atmosphere where they (limit) him, and don't allow him to act, he actually says "Wait a minute, this is what they expect of me", and he stops asking why. Unless I decide to break open the framework and I continue asking why anyway, and then they'll say that I'm hyperactive or something.

Rachel: Those are the first two parameters of MLE. We'll talk about the others as we go along. Each time when it happens that I mediate something in some way, I hope to point it out specifically. But it's possible that you will notice when I am mediating unconsciously. Because all in all mediation is something that we most often do without being aware of it.

(Transcript, 5/11/97, pp. 12-14)

Although these future teachers, as is often the case, had experienced a life-time as children in classes where they were told what to do without being given reasons for their doing it, as soon as I mentioned the importance of mediation of meaning they realized how untenable this practice could be. This transcendence was an effective strategy that I used often - framing educationally sound ideas in ways that connected with their own experience and then having them work out for themselves the reasons that this practice is to be desired. Goni's comment served to further connect the idea of the mediation of meaning with another situation that is familiar to many - young children's tendency to ask the question why all the time, and the consequences that result when the child's environment does not reciprocate when they do. Here, unfortunately, I did not taken advantage of her comment to discuss Feuerstein's idea

of reciprocity on the part of the mediator.

There were a number of occasions when I referred to additional parameters of MLE by relating them to what we did.

> I had them list as many ways of sorting the "defined" treasures as they could, then play the guessing game again . This time the classifier would have to think of attributes that the group had not yet listed - mediation of challenge. We talked about that at the end.
>
> (Journal, 4/11/97, lines 34-35)

In order to illustrate intentionality I used an example of group work from the previous week which I had neglected to mediate during that lesson:

> Rachel: For example, why did I have you work in groups at all?
>
> A few students: Cooperation. I show my way of doing something to a friend...
>
> Rachel: Here I have mediated for you the fact that there are reasons behind my having you work in groups. I have let you understand that this was intentional, that I want you to learn something from the very fact that I had you work in groups.
>
> (Transcript, 5/11/97, p. 12)

This was an early instance in which the keeping of my journal was to aid me in our MLE work. Although I would often forget to mediate a particular point, or neglect to mediate a certain mediation, reflecting quietly on the lesson later on offered me the opportunity to correct the situation during the following lesson.

In the third lesson on sorting I introduced the Attribute Blocks, geometric shapes that are sorted by their shape, size and colour. I had the students work out for themselves what they thought would be in the set.

> As usual, I had them guess what was in the box - first taking out a few of the blocks according to their requests, then asking them to figure out how many there were in all...When we discussed the possible answers to the question of how many blocks there are, I talked about the fact that the actual number was an arbitrary decision of the manufacturer - that they could have been made up of an additional colour, size, etc. I said that the important thing is the children's thinking behind their answers, not their answers.
>
> (Journal, 12/11/97, lines 7-25)

Then I asked them why I had introduced the Attribute Blocks as I did.

> This part of the lesson was where they gave me reasons for my having done something that I myself hadn't thought about. One was that I was preparing the kids for a guessing game; another, that the kind of thinking needed to figure out what was in the box is the same kind of thinking needed to play the game; that it helped learn about the blocks that we would be working with - colour, shape, size.
>
> (Journal, 12/11/97, lines 107-111)

The students' comments seem to all have been particularly enlightening - I myself had never given enough thought to the many reasons for introducing the blocks in this way.

As may be seen, this was not the first instance in which the students had mediated meaning in ways that I had not thought of myself. It suggests the richness that can be obtained when the mediation of meaning is a shared responsibility.

It may be seen that my decision to ask the question 'why" as the main focus of the work in this module encouraged the students' active participation in the attribution of meaning. As discussed earlier, Feuerstein does not consider this way of mediating meaning. For him, it is the mediator who is meant to be the one to mediate meaning for the learner. As a result of my constructivist view of learning, I wanted my students to actively build their own understandings. Therefore, I incorporated an additional strategy for mediating meaning: asking my students to work out for themselves the reasons behind the things that we did.

In the discussion quoted above I expressed my preference that we mediate using questions rather than simply comments. There were other instances where I clearly stated my own opinion as well. Sometimes I felt comfortable doing so; at other times I did not. Often when I felt passionately about a particular issue I tended to mediate my own meaning. For instance, my desire that the students realize the importance of having children *think* in mathematics classes led to a number of instances where I mediated my own meaning rather than leaving it up to them. One of these occurred at the end of the introduction to the Attribute Blocks. This time I chose to specifically mediate the importance that I attributed to the work with the blocks:

> I said that the important thing is the children's thinking behind their answers, not their answers.
>
> (Journal, 12/11/97, lines 7-25)

And a second example was a general comment about the course:

> Rachel: You will see that this lesson is only partially mathematics per se. You'll see that many of the things that we do are connected with thinking because the most important thing that we want them to do in mathematics lessons is to think - as opposed to what often happens in those lessons where the one thing they don't do is think.
>
> (Transcript, 5/11/97, p. 6)

One of Feuerstein's central reasons for mediating meaning is to establish in the mediatee the propensity to look for reasons independently in the future. Buchmann (1986) provides an additional angle by looking at the issue of meaning from a professional stance. She holds that it is part of the role of the teacher to provide acceptable reasons for specific curricular decisions and teaching practices.

> The question, then, is what counts as good reasons in teaching. I argue that in many teacher actions, personal reasons are subordinate to external standards... Providing acceptable justification requires the existence of a community to both set standards for adequacy and determine a set of rules for guidance. The role obligations of teachers as members of such a community forge bonds that not only ensure compliance but generate effort and involvement.
>
> (Buchmann, 1986, p. 529)

It may be seen from the data that, through mediation of my own meaning, I was attempting to encourage the development of this need on the part of my students. As Arianne wrote in her lesson summary at the end of the module,

> The teacher includes her personal taste in what she says, and even notes what she particularly likes and the things that she thinks are worth investing more work in.
>
> (Arianne's summary report, 18/11/97)

Although my stated goal was to allow my students to mediate their own meaning to the greatest possible extent, it may be seen from the above examples that I in fact employed both, possibly slightly contradictory, ways of attributing meaning. In order for the learning to be both cognitively and emotionally meaningful to the students, in accordance with a constructivist theory, I did make a somewhat successful effort to have them be the ones to attribute their own meanings to the issues raised. On the other hand, both to mediate for them the propensity to look for their own meanings as suggested by Feuerstein, and to mediate a professional view of mathematics education, as suggested by Buchmann, I continued in many instances to mediate meaning as I, a representative of the professional community, saw it.

EVALUATION OF THE FIRST MODULE

The Students' Work

Because of the short duration of the first model, only three weeks in which I saw each group only once a week, it was too early to look for significant development and change in the students' thinking to that point. Some evidence of initial development, however is apparent in the exchanges quoted above. It is also possible to look at their participation in the lessons and the ways in which they reacted to them. During the module I was encouraged by the level of participation in our lessons, and with their contributions to the discussions we had on different subjects.

> Listening to the discussion about sorting on the tape...it seems that quite a few people were actively involved, and, thinking back, I have a feeling that most were interested in what was going on...Somebody brought up the point that it will help the kids learn to accept opinions of others - I couldn't make out how it was initially put, but my first reaction was that it's connected with flexibility, and somebody else picked up on that.
>
> (Journal, 19/11/97, lines 90-105)

Tammy, as lesson reporter, remarks on the work of one small group:

> There's an argument between Hilit and Gail. In my opinion every argument is an attempt to mediate. Each one is trying to explain her thinking to the other and trying to influence what the group does through her own logic. That's certainly true in this instance.
>
> (Tammy's report, 12/11/97, p. 5)

Tammy's spontaneous reflection on the meaning of mediation would seem to indicate that its discussion in our lessons has influenced her thinking and that MLE is beginning to serve as a tool which she sees as being useful for analyzing a range of human interactions, not only that between teacher and student.

My Own Work

Successes

From the point of view of my own teaching, I had set a number of goals to be achieved during the first module. The first was to increase my ability to use the criteria of MLE in a conscious and more systematic manner than had been the case until then. The conscious use of MLE, although originally conceived of as a strategy to improve my teaching, in fact proved to be a learning process rather than a strategy that could be "applied" at will. By the end of the first module I was able to see some small successes as well as some problems that needed to be addressed.

On the side of success I was able to see, through an analysis of classroom incidents, that I succeeded in incorporating all three criteria of MLE that Feuerstein sees as being essential to the mediation process. Although the mediation of meaning was the most visible in my work through explicitly posing the question "why" on numerous occasions, an analysis of these episodes showed, as suggested by Feuerstein, the ways in which the question "why" embodies within itself intentionality, reciprocity and transcendence.

In addition to this regularity in my mediation of meaning, analysis of the above data indicates that I was able to maintain a reasonable balance between the mediation of my own meanings and my allowing the students to work out theirs. This was the major way in which I incorporated a constructivist approach to the mediation of meaning. At the time the importance I attributed to this approach stemmed from my concern that the knowledge gained by my students be robust enough to influence their teaching in the future. Rephrased in terms of beliefs, it seems that in order for this goal to be achieved it is necessary for the teaching to be such that it has a profound and lasting effect on my students' basic beliefs about mathematics and mathematics education.

> Influencing belief change has a far greater impact on learning than do direct modes of presentation or instruction. It can be contended that programs that do not explicitly deal with student teacher beliefs will lead to less learning (at least at a reflective, or deep processing level) and consequently will not affect subsequent teaching performance, i.e. the deployment of new concepts in actual teaching in practical settings. It would be appropriate to envision beliefs as guiding the knowledge construction of student teachers.
>
> (Tillema, 1997, p. 294)

Third, I found that, to some extent at least, I succeeded in using the mediation as a

way of causing the learning taking place to be situated in the immediate educational context of the college classroom. By mediating many actual occurrences in our lessons and drawing out their implications for pedagogical learning, I was able to take advantage of this unique educational situation where students who are learning about education take part themselves in an educational context.

In addition to these, I was also beginning to see how the criteria of MLE could be used to evaluate the quality of my teaching. Although in the interests of space and simplicity I have demonstrated this in the above analysis by referring to the three essential criteria of MLE, my reflection was informed by other criteria as well. Together they provided me with an important tool for a more detailed analysis of many of the events which took place in our lessons. Although this may not be seen in and of itself as success in my teaching, it contributed to that success through the reflection that it made possible.

Difficulties

On the problem side, three main issues disturbed me. The first, the other side of the mediation coin, were the problems I had putting MLE into practice. There were times when I simply neglected to mediate. One example of that was when I did not at first mediate the importance of the group work we were doing in our lessons.

> About the group work - I just realized that I didn't even try to mediate it. Group work has become so taken for granted that I never even thought about pointing out its advantages, asking why I do it.
>
> (Journal, 4/11/97, lines 41-43)

And the following day:

> I noticed that I make an effort to accept all ideas that they put forward, and to understand what each of them is saying - as I would want them to do with the kids. But I have not yet mediated this. I should make this explicit, and ask them how they feel about their participation in the lesson.
>
> (Journal 5/11/97, lines 11-15)

And another:

> I see that I make an effort to accept all ideas that they put forward, and to understand what each of them is saying - as I would want them to do with the kids. But I have not yet mediated this. I should make this explicit, and ask them how they feel about their participation in the lesson.
>
> (Journal 5/11/97, lines 11-15)

Neglecting to mediate the mediation

There were many times when, although having mediated, I did not mediate the mediation. There were instances when that occurred as a result of lack of awareness - in the stormy waters of teaching my attention was often elsewhere. One instance of this was:

> Listening to the tape - I had them construct the a-blocks - letting them guess what other pieces were in the box, and after taking out a few, the number of pieces altogether in the box. But I never discussed with them the kind of mediation I was doing - intentionality & reciprocity, challenge, feeling of competence,...
>
> (Journal, 11/11/97, lines 3-5)

There was one example where I focused on transcendence but was unaware of the fact at the time, and therefore neglected to mediate. In this instance I had shown them how the format of the Creature Cards, in which examples and counter-examples of imaginary concepts are given, could be used to teach mathematical concepts as well:

> They seemed to enjoy the first part of the lesson - the Creature Cards - and liked the idea of using that kind of activity to teach all kinds of concepts. That, by the way, was definitely transcendence, but again I didn't mediate the mediation.
>
> (Journal, 18/11, lines 20-22)

But there were other instances where, although I was clearly aware of having mediated, I did not follow through and mediate the mediation.

> When I ask them how many blocks are in the box, they stop and think for a second, with the expectation that they should be able to give me the answer immediately. I stop them and talk about the fact that the question I have asked them is a problem, and being a problem needs to be thought about and figured out methodically. Again, I mediated the idea of a problem (in this case mediation of regulation and control of behaviour) but I neglected to mediate the mediation.
>
> (Journal, 11/11/97, lines 41-45)

When Donna expressed her dislike of the sorting games I consciously used her comment to mediate the situation to the students. This time not only did I not mediate the mediation, but I also did not notice the fact at the time of writing the journal:

> ...most mentioned the challenging aspect of it. But one student, Donna I think, didn't enjoy playing it. I don't remember why now, and couldn't make it out on the tape, but I used her feelings to remind them of the fact that not

> every kid will like every activity - that's legitimate, and it would be necessary for me as a teacher to find a different way of having the child learn this subject through a different kind of activity.
>
> (Journal, 12/11/97, lines 95-100)

On analysis it may be seen that in this case I did not mediate the intentionality and reciprocity present in the situation - "I want you to take notice of what Donna said" - or the request for reciprocity on their part - "and pay attention to it when you work with children" - nor the transcendence in that I compared Donna's feelings to those of children in the classroom.

Possibly because of the difficulty I was experiencing, I was beginning to realize the central place of mediating the mediation:

> That was the main problem with today's lesson - because this is a course in teaching them to become teachers, the main thing I should be mediating to them always is the mediation. I wonder whether this inadequate mediation, whether in quantity or in kind, might not be one of the main factors in the failure of didactics courses to achieve their goal.
>
> (Journal, 11/11/97, lines 41-49)

The mediation of my mediation was the major , most situated way in which to introduce my students to the ideas of MLE, and to demonstrate the ways in which they can be put into practice. My difficulty with this strategy during this module resulted in the loss of important learning opportunities for my students.

Additional problems of the first module

There were two additional interconnected problems with which I was struggling during this first module. These were not specifically connected with the use of MLE, but they affected that as well. One was the pressure that I felt to cover as much of the early childhood mathematics material as possible. In order to do this I made a great effort to complete the material I had planned for each lesson. From as early on as the first lesson with one of the groups I curtailed one of the activities in order to do so.

> When I did the first sorting activity - when I decide on the group and tell them where to stand - I played only one game with them because time was so short.
>
> (Journal, 29/10/97, lines 23-24)

In my journal I wrote:

> I think my main problem in the lessons at this point is the pressure I feel to cover all the subject matter. It doesn't allow me to deal with problematic

> situations as effectively as I might. Maybe I need to rethink my curriculum - I know that teachers of didactics of mathematics for the higher elementary grades have no intention of covering all the material that is taught in elementary school - that's one of the reasons that I like the early childhood programme, that you can more or less get through everything they're going to need in school. But maybe I should consider cutting out a few things, so I can allow myself to teach what I do teach better. I know that's going to be very difficult for me - it doesn't fit in with my perfectionist spirit. But it might be necessary, particularly now when I have to include all kinds of extras concerning the research, as well as taking into account my new view of situated learning, and including activities such as videos and the study of cases...
>
> (Journal, 12/11/97, lines 48-58)

One of the ways that I attempted to 'cover' all the material was through lecturing rather than allowing for the time it takes to have my students build their own knowledge in a more active way. This was the second problem: giving up complete control of the lessons and allowing my students to do more of the work. This was in effect the problem that my planned second action strategy, implementing an extended application of constructivist theory in my work, was meant to address. I had identified my tendency to lecture too much before beginning my research. Although according to my plan I meant to focus on this strategy during the second module, my attempt to have the students be the ones to mediate their own meaning by asking 'why' questions represented a first step in this direction. I had also made some attempts at planning more active, group-based work when working on pedagogical issues that I had previously addressed using a lecture format.

In the second week of the school year, I had already related to this issue. Although I seemed to feel that the lesson had gone well and that I had not lectured too much, the problem still concerned me.

> I still find myself talking too much in the lesson. Although I do give them lots of opportunities to talk, and when they volunteer to talk on their own, I always listen to them, and often let them speak fully. But sometimes they say just a word or two that reminds me of something I want to say, and I begin a long explanation of my own. The things I have to say I know are important, and from the students' reactions I know they find them interesting. But the net effect is that I talk a lot and they talk too little.
>
> (Journal, 5/11/9, lines 43-48)

Two weeks later I identified an opportunity that I could have used to have them do the work themselves, but from force of habit neglected to do so:

> Listening to the discussion about sorting on the tape...The fact is that I could

> have asked them to expand on the idea - I'm still used to expounding, so the possibility of having them do so does not yet come automatically. That clearly would have been the better way to do it
>
> (Journal, 19/11/97, lines 90-116)

Part of the problematic nature of making the shift becomes clear in my journal entry after having given a lesson in which I did change my teaching strategy. In this lesson I introduced Piaget's classification of knowledge in which he divides knowledge into three categories (Kamii, 1985): physical knowledge, logical-mathematical knowledge, and social knowledge. Rather than describe each of the different kinds of knowledge as I had done in the past, I presented the names of each and asked them, in groups, to try and figure out what each might be referring to.

> I wasn't aware that the use of the word "Physi" in Hebrew would be problematic - it sounds to them like it has something to do with physical activity or fitness, and that mixed them up a bit. I realize, though, that giving them that kind of activity requires much more knowledge on my part - because I have to respond to what they say, and feel comfortable with what I know. It's a question of being prepared for unexpected things. That's what I've been thinking about regarding subject-matter knowledge, after all. This is a perfect example - much easier to present a subject, say what I'm prepared to say, possibly field a few questions, and that's it. In spite of the fact that I know they often or always can learn better this way, I'm left with a feeling of dissatisfaction because things were never said succinctly and well - it's much more diffuse thus way.
>
> (Journal, 18/11/97, lines 42-33)

Although generally satisfied with the way I had introduced the material, I nonetheless felt uncomfortable about not being able to deal systematically with the subject matter and know that I had "covered" it as I might have done in a lecture:

> I started the lesson with a discussion of Piaget's physical, logico-mathematical and social knowledge. After introducing the names of the different kinds, I had them discuss in groups what they thought those names might mean...Again I felt the loss of control that comes from letting them figure things out for themselves.
>
> (Journal, 19/11/97, lines 3-7)

In spite of my attempts to let go of the reins and allow the students to take over the major role in our lessons, I still had a strong predilection to lecture. This situation did not show signs of changing quickly. The problem would need to be addressed more intensively in the second module.

Summary

The importance of my awareness of the use of MLE lay in the fact that it offered three clear advantages to my teaching: the control that it allowed me to have over the quality of my teaching, the useful analytical tool that it provided in evaluating my teaching, and the opportunity it presented of mediating my mediation for my students. Focusing on the mediation of meaning in this module provided me with an easily applicable strategy, the use of the question 'why', which could introduce my students to many of the central ideas and issues to be raised in the course of year. The many instances in my journal where I used the concept of mediation to look critically at my work offered me a fruitful angle from which to judge it. A large proportion of those journal entries related to my lack of success in mediating the mediation in those situations where I did successfully mediate. Although the first module had come to an end, I was far from having successfully put into practice my first action strategy. That action cycle would have to continue into the second module, along with the beginning of the implementation of my second strategy.

CHAPTER SIX

THE SECOND MODULE - NUMBER SENSE

The subject matter of the second module included the development of basic numerical understandings and skills many of which, while obvious to adults, are not at all obvious to young children. As discussed above, the problematic nature of teaching this subject matter to adults was the stimulus that had led me to embark upon this research project. In the past I had attempted to teach this material to my students in the same way as I had done with mathematics material meant for the higher elementary school grades: by having the students experience the same activities and ways of thinking that children experience when they learn mathematics in a constructivist classroom

However, I soon realized that a model of mathematics teacher education which encourages the construction of mathematical knowledge by adult students in the same way as children, would not be appropriate for much of the material required in an early childhood curriculum. I needed to find teaching strategies consistent with my constructivist perspective that would address the content particular to a teacher education course: the pedagogy associated with mathematics education. This meant that rather than basing a lecture, or a whole-group discussion, on previous experiential mathematics activities, the actual work on pedagogy had to become active and experiential in its own right. The application of MLE theory, which looked at ways to introduce work to students that would ensure their active participation in the learning process, was one way in which this could be done. Looking for constructivist-inspired strategies to relate to pedagogy was another. The latter goal was the focus of this second module. I first present the data in relation to it, before returning to a consideration of the MLE as it manifested itself in this module as well.

The overall action strategy that was to be the focus of this module was to teach pedagogy "constructivistically". The strategies devised for this endeavour can be divided into three categories: 1) allowing students to actively learn what I had previously taught by lecturing; 2) taking advantage of their written assignments to encourage the construction of their own understandings; and 3) explicitly working on the students' development of their own educational principles.

Teaching Pedagogy "Constructivistically"

The Development of Number Sense - Basic Mathematical Understandings

The first lesson of the module dealt with basic numerical understandings of children - their ability to establish one-to-one correspondence between the objects of two groups (i.e., one plate for each child), their ability to see the relationship between the parts of a whole and the whole (that a group of 5 children is at the same time a group of 2 girls and 3 boys), their ability to conserve quantities (a set of 8 objects continues to be made up of the same number of objects, no matter how they are arranged), and their ability to count and understand that the last number that is counted in a set represents the number of objects in the whole set.

Although I presented those in lecture form, I used a number of strategies to stimulate the students to think more actively: I attempted to use an element of surprise regarding children's unexpected understandings, I demonstrated the points as concretely as I could, and I warned them that the lesson would be entirely lecture and whole-group discussion and asked them to consider why I decided to teach it this way.

The creation of disequilibrium

Piaget's (1971) discussion of the development of knowledge, in which the subject assimilates information into his or her knowledge structures through a process of equilibration, shows how the individual constructs new knowledge based on these previously-existing structures.

> Learning in terms of experience is therefore not due to pressure passively felt by the subject but to the accommodation of its assimilation schemes. A certain equilibrium between assimilation of objects to the subject's activity, and the accommodation of this activity to the objects thus forms the point of departure of all knowledge...
>
> (Piaget, 1971, p. 108)

Flavell, in his treatment of Piaget's stage-independent theory, asks the question "What are the general principles by which the subject...changes his state in the course of development?" He calls this question the "diachronic" (as opposed to "synchronic") question.

> Across a childhood of continuous operations of the functional invariants, arise a succession of discontinuous cognitive structures. This is the heart and essence of cognitive development...The sequence of cognitive structures becomes, in this interpretation, a sequence of equilibrium-state "moments"

within an ongoing, continuous process of equilibration.

(Flavell, 1963, p. 263)

In the first lesson of the module I brought up a number of conceptions that children have about number that are different from those assumed to be obvious by adults. One of these is the conservation of number discussed by Piaget in his work. Another was children's lack of cardinality, the understanding that the last number said when counting a group of objects shows the number of objects that there are altogether in the set (Gelman & Galllistel, 1978). And third was the children's difficulty in seeing simultaneously the whole and its parts: the understanding that 5 cookies, divided between two plates, are still the same five cookies (Kamii, 1985).

Having shown the students one test for conservation of quantity, I then asked the question:

> What do adults usually think when children answer their questions 'incorrectly'? - they think that children just don't know the correct answer, and that they need to be corrected. But *from the point of view of the children, they are right.* From previous experience children know that generally, when one row of objects is longer than another, it is because it contains more objects than the other row. And if adults correct them, it's like pulling the rug out from under them - the adult is in danger of causing the children to mistrust their own judgment and to accept uncritically adults' explanations rather than work out independently their own understandings and explanations.
>
> (Transcript, 26/11/97, p. 2)

As may be seen, there were a number of surprising points in my explanation, among them that it may be adults' short-sightedness and lack of understanding that lead them think that there is no logic in children's answers, and that their tendency to correct the children's answers may do more harm than good. This point of view had the potential to make my students think again about assumptions that they previously had no reason to question.

Cardinality refers to the children's understanding of the counting process. Much to the surprise of many adults, many small children count without having any understanding of the reason people generally count - to know how many are in a set. They are not aware of the fact that the last counting number points to the sum of the set of objects being counted, and the only reason that many of them perform the counting act is that this is what they have learned to do when adults asks "how many?".

In order to demonstrate this point about cardinality, as well as other principles of counting discussed by Gelman and Gallistel (1978), I had asked the students to observe carefully while I slowly and deliberately counted a set of objects, and then asked them to analyze the operation that I had performed. Maureen reported on the lesson:

> Rachel counts shells while the students watch.
> (She asks) What did I do?
> She summarizes the analysis of what she did on the board:
> I touched and moved (the objects)
> I said the numbers
> I counted rhythmically
> I used one-to-one correspondence - one number for each shell
> Here Rachel shows us actions that she does in order to count . She is teaching us the material in a concrete way and summarizing on the board. I think the students understand better this way...
>
> Rachel gives an example of a child who makes a mistake in the counting and asks the students what is happening with him. (He doesn't know that the last number counted is the amount of objects altogether). The students cooperate and are interested in the reason.
>
> (Maureen's report, 25/11, p. 3)

As expected, the students did not figure out the cardinality principle on their own, and I needed to direct their attention to this aspect of counting. Maureen's report reinforced my own feeling that I had succeeded in taking advantage of the disequilibrium created in their understanding of children's counting to involve them in attempting to understand the reasons behind the phenomenon

The third surprising point was the difficulty that children have in simultaneously seeing both the parts and the whole (Kamii, 1985). For example, to an adult one ten is obviously ten ones. To many children this is not at all obvious. This limitation in children's capacity to deal simultaneously with two different factors at different levels of generality is extremely difficult for many adults to understand. Sherry, the reporter, writes:

> Rachel says that small children have difficulty seeing the parts and the whole simultaneously. Rachel asks one of the students if she understands the way children think. The student answers: Understand? No. Trying to? Yes. The student is trying to understand what happens with children, how young children think when they are taught the base-ten system.
>
> (Sherry's report, 26/11/97, p. 3)

Sherry's report reflects the difficulty that at least one student experienced in understanding the way youngsters think as well as the efforts they were making in order to do so.

The mediational quality of the interactions in this lesson may be seen by referring to Feuerstein's view of the effects of mediation on all three participants in the interaction (Feuerstein & Feuerstein, 1991). As referred to in the discussion of intentionality and reciprocity (Chapter Two), these participants are the mediator, the mediatee, and the object to be mediated. In this lesson I attempted to present the phenomena discussed as dramatically and surprisingly as possible. In this way I altered both myself and the subject matter, thus influencing the students' attention and their attitudes as well. On analysis it may be seen that in this way I also encouraged them to think critically about their previous perceptions of children's mathematical understandings and abilities, and of the teacher's role in their learning, in this way stimulating them to mediate their own meaning.

Again asking why

The lessons of the first module which had been so different from traditional college classes seem to have provided the background for a response to the question of why the lesson had been taught in this format, called "frontal" in Hebrew.

> Enid: Because there are some things that you can't explain by having us try them out. With sorting you could give us a feeling for how important it is by having us try out the activities.
>
> Rachel: And what was it about these things that made that impossible?
>
> Enid: Because we already conserve quantity and know how to count, so we can't understand what it's like not to know.
>
> (Transcript, 2/12/97, pp. 7-8)

Although there seemed to be general understanding regarding my reasons, a few of the students expressed their clear preference that there be some activity in the lesson as well. In my journal I reflected on what they had said:

> When I asked for their reactions towards last week's frontal lesson, one of the students expressed her clear preference (and mine too) that the whole hour and a half should not be frontal. Although there were others that felt otherwise, who felt that they were active enough in the lesson to keep them interested and learning, I now feel that if I know that so much is frontal, I should have at least given them some activity which would make them feel the difficulty of learning how to count - like teaching them to count in a foreign language, for instance, or getting them to start from the middle word of a well known poem.
>
> (Journal, 2/12/97, lines 74-80))

Both the fact that I had felt there was no alternative but to present this material in lecture form, and the fact that in my journal I regretted not having given them an activity that they could do as children might, indicate the difficulty I was still having at that point devising a more constructivist-inspired, active approach to the teaching of pedagogy. During the discussion I said, in reaction to their comments,

> It was just too much. So take that into account in your own teaching. If for you one and a half hours (of frontal lecture) is too much, then for children fifteen minutes is too much. You have to take your own feelings as a student into account and make the direct connection with the work that you will do with children.
>
> (Transcript, 2/12/97, p. 2)

Here I was hoping that my students' experience of my own "failure" and the fact that I mediated that failure so they would be aware of its shortcomings, would have an effect on their own teaching in the future. It is possible that the difficulty which some of them felt with this format would reinforce their growing understanding of the effectiveness of learning based on greater activity.

The Development of Number Sense - The Whole and Its Parts

In the following lessons we began to look at number as the whole and its parts. Two of the most important ways in which children can begin to develop a number sense are through counting and estimating quantities. After having them take part in an activity where I flash a set of objects for a second or two and then have them estimate the amount, I had them work out for themselves the different kinds of thinking and acting that are involved in counting and estimating:

> An interesting point was brought up... They said that counting is largely technical, using more social knowledge rather than logico-mathematical, while estimation uses more logico-mathematical knowledge. It's actually another way of saying what I had in mind, that estimation requires more thinking, more evaluation of the situation than does counting...I brought up the criterion of mediation called mediation of regulation and control of behaviour - that counting calls for slower, more careful and controlled behaviour, while estimation is a quicker process which does not call for that much care - it sounds like a paradox that the activity that requires more logical thinking is the one that requires less care and time, and vice versa.
>
> (Journal, 3/12/97, lines 11-19)

The students' understanding of the essence of the two activities seemed to develop from close attention paid to the essence of estimation which they had just experienced, and to that of the counting we had done the previous lesson. In addition to taking into

account both of these, the use of Piaget's classification of knowledge (Kamii, 1985) to analyze the difference between the two also seemed to show the effectiveness, for at least one individual, of the activity that we did at the end of the first module. The connection made between the three activities provided an excellent opportunity for the class to reflect on the distinction between counting and estimating as understood in Piaget's categorization of knowledge . Evidence of the success of the three activities that dealt with these ideas, was seen in at least one student's insightful analysis of the two operations. In this instance the connection I made between the components of counting and estimation and a specific criterion of MLE may have extended this insight, but the major work was done by the students themselves.

Estimating is one logico-mathematical operation that causes some people to focus on the parts in order to estimate the total amount. We often look at the more easily perceivable parts and make conclusions about the whole in accordance with our perception of them. A similar understanding is necessary in order to compute exact amounts. Through a series of activities I had the students experience the ways in which children can be directed to take notice of and remember the various combinations of numbers that compose the numbers from 1 to 10. For example, if we take six objects, they can be seen as two objects and another four, one and five, two and two and two, two times three, or six and zero. The first lesson in this series, although including two instances of more active work, was for the most part again run as a whole-class discussion/lecture. This for many of the same reasons as those in the previous lesson.

> I told them rather a lot in this lesson - ... deciding on what the purpose of a lesson is and working toward that purpose, not wasting time and energy on doing something that has no connection; the meaninglessness to children of written computation, how it doesn't usually build on their previous knowledge; ...and finally on the advantages of choosing one number to work on at a time, learning all the different ways to form that number, so they have a chance of remembering what they have done...I also went off on a tangent about teachers that are afraid of mixing children up - who will only use one material to show a particular concept, or who won't give kids tasks that are too difficult for fear of having them not understand. That was a whole additional "lecture".
>
> (Journal, 10/12/97, lines 4-24)

My comment "I told them rather a lot in this lesson" once again reflects my discomfort with this state of affairs. On the other hand, in terms of offering alternative ways of looking at mathematics education, these lessons seem to have been particularly rich. Nonetheless, my skill at finding ways to present pedagogical subject matter in a more

active manner was in the process of developing, and much of the work done in the second module was beginning to reflect this development. One instance was in the work that we did on the number stations (Baratta-Lorton, 1995). I wanted the students to be aware of the many opportunities for mediation that arise in this activity. Here one number is decided upon, a different material is placed at each table, and the children are asked to build various shapes that are all made up of that number of objects. When the "exhibits" are ready, all the children are invited to a tour of the "museum" and are asked to think what the different "works of art" remind them of. After this initial tour of all the exhibits, a second tour is done, but this time the children's creations are described in terms of numbers: "I see three blocks here and another five blocks here"; "There are four on top and four on the bottom"; "This one is just a row of eight squares".

The lesson with the first group was structured in the "traditional" way: After having them take part in the activity I showed them the opportunities for mediation that were built into it. After reflecting on this lesson in my journal, however, I modified the lesson for the second group:

> This time I wrote the names of the different criteria that I found on the board, and I asked them to think of ways in which each criterion may exhibit itself. That was better than what I had done the previous lesson when I had just told them what I had seen.
>
> (Journal, 10/12/97, lines 48-52)

Some of the responses were:

> Rachel: How do you see the possibilities for transcendence here?
>
> Nadia: Each one sees the shapes in a different way.
>
> Miri: I built something (not clear)
>
> Rachel: Good. First of all it's something that *I* built - in that way it's also 'individuation and differentiation'. I did it like this, she did it like that. And what else is important here? That people have different views of things, different ways of viewing things - and they're all legitimate. It's possible to listen to others and understand their ideas. This idea transcends by far the actual activity....
>
> Now how about the mediation of a feeling of competence? How could that fit in here?
>
> Nadia: Because here everyone is capable of building something.
>
> Michaela: But if a child doesn't want to? If he feels that he can't build anything?

Rachel: That's why it would be important to mediate here. This situation is one in which it should be possible for everyone to build something they are pleased with.

Nadia: What's important is that ... there are materials here that the children can touch and manipulate. And then they can use their imagination.

Hadar: I'm not here to judge what they've done. I'm only looking at what the shape reminds me of.

Rachel: Everybody's looking at what they did, they are interested in it, so the children feel good about what they did. Also regarding the mediation of a feeling of competence - there are children whose left side of the brain is more developed and there are children whose right side of the brain is more developed... I include in the activity different abilities - you don't have to be a particular kind of person to succeed in this.

Miri: You provide everyone with an equal starting point.

Michaela: Especially with little children because it is something that they do all the time.

Rachel: Regulation and control of behaviour - how could that come in?

Nadia: I build something with the material - myself, on my own - and then I wait patiently to see what the other children see in what I've done.

(Transcript, 10/12/97, pp. 3-6)

From the transcript it may be seen that , in spite of my feeling that I was taking a too active part in the exchange, a number of important points were brought up by the students: the feeling of all children that they are capable of doing the work that is asked of them, the fact that each child sees the same thing in different ways and the legitimacy given to this, and the patience and willingness to hear what others have to say about their work.

The Development of Number Sense - Games

After the work on these number activities we began to look at the subject of games, the second component of the number sense module. The subject of games was one that I had been thinking about quite a lot. I had begun to work out for myself the characteristics of what I considered particularly good games - those that not only give children multiple opportunities to engage in numerical computation, but have additional attributes to recommend them, such as the need to think mathematically, logically and/or strategically, the need to cooperate with others, and the need to discuss and verbalize their thinking. I was interested in helping my students develop a critical

view of games, analyzing their advantages and disadvantages and making well-informed judgments about which games to introduce to their pupils. I wanted to encourage them to consider the potential inherent in different games and to become aware of the ways they might mediate in order to take best advantage of them. When I first encountered difficulty in generating the kind of critical approach to games that I had hoped for, I initiated an action research cycle which allowed me to reflect on the problems I identified, to plan and implement action strategies, and to observe and evaluate their effects. Although this work was done with both of my classes, as was generally the case, there seem to have been much more encouraging results with the Wednesday class than there were with the Tuesday group. It is possible that part of the reason for this was that, this being a short action research cycle, the second class of the week had the advantage of teaching that was informed by my experience of the day before. In addition, at that point in the year I was generally experiencing difficulty with the Tuesday group.

> Lord help us! I'm afraid the lessons with this group are getting worse and worse, for whatever reason. It might be because of the time that has been wasted because of [those that had been opposed to the research]. Or it might be because of the reception that my suggestions get - not full of enthusiasm from the students - lots from the in-service teachers, at least until now.
>
> (Journal, 23/12/97, lines 3-7)

When I first introduced the subject I gave them a few games to play, and then asked them to consider the advantages and disadvantages of each one. Some of the students' comments were: "For children this game will be really good. It teaches addition with the help of dice, and in an enjoyable way.", "The game looks boring to me.", "There is always someone who loses.". And for another game: "In my opinion it will be difficult for the kids. But it's a nice game.", "It's a really irritating game." And a third game: "It's a nice game but the children I work with won't like it." "It is on a high level and necessitates a lot of thinking and time. It might go very slowly." "It would be better to start with fewer pieces in order to focus their thinking." Some of these responses were well-thought-out, while others were overly general and purely instinctive.

> A few times some of the students mentioned how boring or irritating some of the games were. That made me realize that I should have emphasized, before we started playing, that these games were meant for children, not planned with adult students in mind. Not that their comments might not be right, of course...
>
> (Journal, 10/12/97, lines 103-106)

I decided to have them play a number of games which did not have any of the attributes that I considered as being important.

> I originally had thoughts of giving them some dry fact-practice games to play as well, and this reinforces that idea. If I want them to come up with the advantages of the good games, the best way is to give them something to compare them with, rather than, again, trying to tell them about what's better and why.
>
> (Journal, 10/12/97, lines 103-119)

At the same time I clarified the goal of these lessons on games, mediating my belief that as teachers they need to learn to be aware of and understand the reasons they have for introducing any particular game.

> I told them about what my goal is regarding the games - that I want them to get to the point that they're able to decide what principles of good games are important, and to be able to design and judge games according to those principles. There seemed to be general agreement and understanding of the importance of that idea - that, in spite of Liron's comment earlier on that every game is good - if children enjoy a game they will learn from it. I told them I was telling them this in response to the feedback I had gotten from them re their wanting to know what the point is of what we're doing. One thing that I do think that came out of this lesson is the extent to which I take them into account in planning my lessons, and the extent to which I really learn from them and from their comments. This really is a good model for them - I wonder if it will have any effect on the way they are with their children in their classes to be.
>
> (Journal,16/12/97, lines 44-54)

In this instance I felt that my intentionality had been reciprocated by the students. In my comments to them I showed that I, too, had reciprocated by taking their feedback into account . Also, in the following excerpt, I referred again to feedback I had received from them which prompted me to suggest a different way of playing one of the games, and took advantage of the situation to remind them once again of the importance of asking why:

> Rachel is explaining to the group how to play the games. She notes that she modified the game in accordance with the feedback she had received from one of the other groups.
> She says: While you are playing, pay attention to why we are playing this game in the first place.
>
> (Ronnie's report, 16/12/97, pp. 1-2)

The use of comparison between "good" games and less-worthwhile ones, as well as my use of MLE, seem to have had some effect, and work done in some of the small groups in the Wednesday group proved to be at a very high level. One example of this led to an important mathematical discussion:

> Some interesting things came out of this group work, although maybe half of the groups needed my help to get going...They brought up the question of what I mean by part/whole to begin with (my first reaction, obviously, was to think oh, no, they haven't been listening - but when the discussion got going there they raised some very basic and important issues which I had never thought of - maybe I think I give them credit but I don't give them enough!)....The most interesting thing that came out of the discussion was the idea of the necessity of defining each time what the whole is and what the parts are. There is a tendency to think, as did one group in this lesson, that the parts are changeable but the whole is permanent. But in the discussion we saw that the whole can become part of another whole. We discussed the problem of kids becoming stuck on particular ways of looking at things.
>
> (Journal, 17/12/97, lines 41-55)

This excerpt shows both the level of the group discussion as well as my continuing doubt that the group work can actually be at a level high enough to "replace" the things I would tell them if I felt that to be an effective strategy.

In my journal entry regarding the following Tuesday class, I wrote:

> I still am not pleased with their handling of the conclusions that they come to after having played the games.
>
> (Journal, 23/12/97, lines 24-25)

Therefore, in another attempt to improve the level of the discussion in the group the following day, I wrote up specific instructions (see Appendix F) for the small groups to refer to after playing the games:

> Try and relate to the games from three points of view:
> 1)What mathematical subject matter do they deal with? What is your opinion of the way this subject matter is presented in each specific game?
> 2)What educational and social values (not mathematical) are conveyed in the games you played?
> 3)What makes a game good? - from the point of view of content area, from the point of view of social interaction, from the emotional point of view?
>
> For the purposes of this discussion you can take into account the games that you felt were most noteworthy and attempt to analyze the reasons for their success. It is also possible to look at the less satisfactory games and try and understand why they are the way they are.
>
> (Lesson Plan, 24/12/97)

It was too late, unfortunately, to use this additional strategy with the Tuesday group. Looking at the work done with the Wednesday class, however, three action strategies that I developed during the cycle can be seen: mediating the focus of the work they were doing on games, comparing the attributes of dry practice games to those of games

that demanded thinking, and writing specific instructions to focus their thinking. Each seems to have had some effect on the students' work, and at the end of the three weeks the responses of the students indicate that progress had been made. This is shown by two examples from the group that was observed by the student reporter:

> The Coconut Tree:
> an excellent game. Mathematically challenging in the computation and the number bonds it works on...
> The message is: "The more money you have the better and more successful you are". We are ambivalent about that. It's a fact of life not to be satisfied with too little.
> The game teaches them to pay attention and to use their better judgment - each player plans his own route. The trading is useful in life.
>
> The Rainbow Game
> It uses a different kind of thinking than usual. You have to get to the end slowly as opposed to the other games. It is aesthetic, and it is very enjoyable. The game is calmer and is not dependent on too much previous knowledge. You don't get too upset by losing, in spite of the fact that it is somewhat challenging...
>
> (Dekel's report, 24/12/97, pp. 1-2)

In addition to these, some points mentioned in the whole class discussion seemed to reflect the process the students had undergone. They also show a greater focus that may have been engendered by these final written instructions. They concluded that good games give the children a feeling of competence which results when they can direct what happens in the game; that they deal with important mathematical ideas (such as exchanging ones for fives or tens, or seeing the connection between addition and subtraction); that they encourage cooperation and discussion; that they make it necessary to think; that they encourage flexible thinking; and that they are challenging. Regarding less successful games they noted that they are too competitive; that they put too much pressure on the children; and that too much depends on luck.
The emphasis in these remarks on thinking, on cooperation and on challenge may indicate progress when compared to some of the earlier comments that had been recorded in this group, that the games were boring, irritating, enjoyable, too difficult, or not fun. As mentioned, similar progress was not observable in the Tuesday group.

Assignments

My goal in giving the students homework assignments was two-fold: to provide them with learning opportunities, and to assess their understanding so that I could plan my teaching in accordance with their needs. In the past, the way in which I tried to further

their learning was by writing long and detailed feedback on their work. This year, however, because of my emphasis on having them construct their own knowledge, my comments were much more limited, and were generally phrased as questions. When I felt that a student could potentially have benefited from the activity by reflecting more deeply, I returned the assignment for correction.

> I started off talking to them about how I want them to report their various assignments - that I don't want a description of the content, but rather reflection, while considering the main issues that I asked them to look at when I gave them the assignment. In both classes today there was unhappiness about my giving them back work for correction. I managed to stand my ground by appreciating their difficulty (in terms of the amount of other work they have to do) and by emphasizing that the point of what I give them to do is to learn - by putting out just a little more effort they can learn much more. I'm wondering now if this isn't connected to my belief in them as changing human beings - that I don't just give them marks and decide that they are more or less successful but I believe that by putting out a bit more they are all capable of learning and changing. This is probably a lot of different kinds of mediation - certainly intentionality and reciprocity, meaning, transcendence, feeling of competence, regulation and control of behaviour, mediation of goal seeking, goal setting and goal achieving, challenge... Wow! Did I ever do the right thing this time!
>
> (Journal, 17/12/97, lines 13-28)

My attitude towards their assignments, which saw them as a further learning opportunity, was at odds with the students' beliefs that they were used as a way to evaluate their performance in order to assign them marks. In spite of the fact that we decided that this course would be self-marked, with each student giving herself a mark I would accept if I felt it was within the bounds of reason, and in spite of the fact that I did not give any marks at all to individual assignments, it was necessary for me to further mediate the meaning I attributed to the assignments: that they were meant as a way to learn, not as a tool to be used to assign marks. I felt that this modeling of good educational practice was crucial if I hoped to achieve one of my goals regarding their future practice - that they refrain from assigning marks to young children's work in mathematics. The belief that mathematics is a subject for which marks can be assigned objectively by simply counting the number of mistakes, that this practice actually reflects children's mathematical knowledge, and that it is one that can encourage children to invest more energy in their mathematics learning, were all beliefs that needed to addressed. One of the ways that I did so was through modeling a different way of looking at their assignments, and by encouraging them to consider alternative reasons for having learners carry out assigned tasks.

Principles

The importance of having my students develop the habit of asking why lay in the potential that attitude has for encouraging mindful action - their reasons for deciding to teach in a certain way, and to choose particular kinds of activities as opposed to others, needed to centre more on pedagogical considerations and less on convention, convenience, or on behaviourally observable content goals. It was not enough for me to expose them to useful curriculum ideas - they needed to understand for themselves the principles that lay behind them.

> Without adherence to first principles, surface procedures tend to be adapted and ritualized in such a way that they cease to serve the "thinking" function they were originally designed to foster.
>
> (Brown & Campione in Franke et al., 1998, p. 68)

In accordance with my desire that they take an active part in the mediation of the meaning of the new approach to mathematics education that they were being introduced to, therefore, after an initial period in which, as described, I frequently explicitly encouraged my students to begin asking why, while at the same time explicitly mediating the principles on which I based my own teaching, I provided them the opportunity, at intervals spaced out during the course of the year, to consider and express those principles that they considered important to them.

In the activity done on the development of principles during the second module, I had the students, working in small groups, discuss those principles that they felt were of primary importance for them.

> As a teacher, I have principles according to which I want to act. My goal is to get to the point where you will be aware of your own principles, principles on which you want to base your own teaching. That way you'll be able to try to match your teaching as closely as possible to your principles - so that you can do what you want to do.
>
> (Dahlia's report, 2/12/98, p. 6)

After initially having them discuss possible principles in small groups, I asked them to then choose and rate, in order of importance, those principles that they felt were essential to put into practice if they were to feel comfortable about the work they were doing with children. These they were to write in their notebooks and save to compare with activities of the same kind which were planned for later on in the year. This was the first time they were asked to consider and express their general belief systems regarding mathematics education. It was intended to emphasize a number of points: first,

the importance and legitimacy given to the individual in deciding on those principles that are important to her; second, the necessity of prioritizing which to attend to and put into practice first; third, to bring to their awareness that although we may have principles we do not and cannot always put them into practice; and finally, the importance of making an effort to put them into practice nonetheless.

From an analysis of the lists of principles developed by the groups, it may be seen that the great majority of them, more than 50%, referred to the structure of the lessons. Some of these reflected often-discussed issues that come up in our lessons, such as "free play with materials" "discussion - whole group, small group, and summary", and "lessons using games", and some reflected concerns with which they had entered the course to begin with, such as , "no more than five children in a group", "planning the amount of time for a task".

Approximately 20% had to do with criteria for worthwhile activities - "division of the children into groups after consideration of and in accordance with the task", "directing thinking by asking questions", "using games as a teaching tool", and "making sure the children are involved in the lesson". All of these criteria were ones that had been emphasized in our lessons during the course of the year.

About 25% considered pedagogical issues - "the development of thinking", "to let them feel the materials and gain experience with them", "to expose the children to a variety of activities", "flexibility on the part of the children", "allow children to develop their thinking and imagination". All of these considerations were ones that had been discussed in our classes. There was one additional consideration, however, "positive feedback", that I had actually attempted to have them think more critically about, but was not reflected here.

Summary

In this module I put the emphasis on my attempts to teach pedagogy in a way that would encourage my students to be actively involved in the construction of their own pedagogical understandings. The emphasis here was on knowledge, the pedagogical content knowledge that Shulman (1987) refers to. After having made explicit their original intuitive beliefs through the mediation of meaning at the beginning of the year, and having provided the stimuli necessary for them to question and critically examine their existing beliefs, the emphasis on knowledge at this point of the year hopefully

allowed them to base their new developing beliefs more solidly. In addition to this, similarly to the way in which I had dealt with the mediation of meaning in the first module, the fact that they were actively developing their own knowledge, rather than being spoon-fed mine, meant that the process was their own, that they had a stake in their developing knowledge, and that these ideas might eventually be incorporated into their own personal belief systems. The activity in which they discussed educational principles was a way in which they could verbalize their developing ideas and consider their relative importance in their own eyes.

Of the different strategies that I employed, my attempts to play a less central role in our lessons, while possibly being the most fundamental, seemed at that point to be the most difficult to put into practice. While students' reactions and comments made me feel that my attempts were somewhat successful , I was repeatedly dissatisfied with the extent to which I talked and the extent to which I facilitated the students' active role in their own learning. The difficulties that I experienced in changing the way I had previously presented the pedagogical content of our lessons were many, and I needed to continue my development in this direction so that my teaching would resemble more closely the theoretical principles which I felt should guide my work.

MLE in the Second Module

My Own Mediating

The data for this module show that I was mediating quite consistently throughout. During my interactive teaching, however, much of the mediation was largely unconscious. In most instances, therefore, I did not mediate the mediation. The time-ordered event listing of the eleven lessons in this module shows that I only explicitly referred to mediation in four of them. One possible explanation for this failure to mediate consciously is that, as I mentioned above, the use of MLE as a teaching strategy is an ability that must be developed over time - intentions are necessary but not sufficient, and, as in any learning process, enough time must be allowed to develop the ability before it becomes a natural part of one's practice. An additional explanation that is connected with this is the turning of my attention to the implementation of constructivist theory. It seems that it was extremely difficult for me to keep my attention on these two important aspects of my teaching simultaneously, and that my decision to put constructivist teaching in the forefront necessarily came at the expense of the use of MLE.

In spite of this, however, there is one striking point that emerges from the data, and that is the way in which mediation often augmented the effectiveness of teaching strategies I had devised to increase the constructivist nature of the teaching of pedagogy. In the work on the number stations, when I had the students actively find the ways in which it would be possible to mediate, my role in the discussion was to mediate, when necessary, the ways in which their insights actually reflected the parameters that we were discussing. In our lessons on games, the lesson structure which was intended to encourage the students' construction of their own knowledge needed to be augmented by effective mediation on my part. At first I tried to focus their thinking by pointing out the reasons that I had them take part in the activity, and later I felt it necessary to further mediate by preparing specific guidelines regarding the focus of the group work they did.

This finding, which on second thought seems obvious in that that would seem to be the whole point of mediation, was originally only vaguely apparent to me. Intuitively I had felt that all three of my wider strategies - using MLE, teaching "constructivistically", and situating the students' learning - were intimately connected one with the other. But an analysis of the work done in this module clearly shows the way in which this could be seen in my work.

My Teaching of MLE

M plan in this module was to begin to introduce the students to additional parameters of Mediated Learning Experience, those that are often present in various mediated interactions but are not prerequisite to the existence of MLE as defined by Feuerstein (Feuerstein & Feuerstein, 1991). The number stations activity provided opportunities to work on many of these. In both of these lessons the analysis of the activity in terms of MLE was planned into the lesson ahead of time. In the extent to which I worked according to plan, this mediation was done consciously and therefore could be and was mediated as well. The work on games was not planned in a similar manner. The result was that although my interactions with the students had a decidedly mediational character ("Think while you are playing the games what they are good for", "What educational and social values are conveyed in the games you played?"), I did not explicitly refer to these as instances of mediation. Mediating my own mediation was the most situated and meaningful way that I could aid my students' growing understanding of the different parameters of MLE. The difficulty that I had in doing so resulted in the loss of many opportunities for learning. It would seem that at this stage, in order to

successfully both use and refer to MLE, incorporating it into the lesson plan was essential.

MLE as an Analytical Tool

It would seem that my most successful use of MLE at this stage of the year was the use I made of it in order to analyze my teaching in the writing of my reflections, and in planning subsequent lessons. In reflecting on an instance where I reminded the students of work we had done on another subject, I wrote:

> This is some sort of transcendence, I would imagine - always making connections between different subjects or different points of view.
>
> (Journal, 23/12/97, lines 108-110)

And as reported above, when I looked at the ramifications of my decision to have the students correct their assignments, I used MLE to do so. Having done so, I gave myself high marks for my decision:

> Wow! Did I ever do the right thing this time!
>
> (Journal, 17/12/97, line 28)

These quotes show how the MLE parameters not only helped me to analyze the occurrences in the lessons, but also allowed me to gradually deepen my understanding of the criteria themselves.

EVALUATION OF THE SECOND MODULE

The Students' Work

Towards the end of the module, some progress was beginning to be evident. Particularly in the series of lessons on games, there was a significant difference in the quality of some of the groups' analyses after I had taken care to provide them with situations which would facilitate their thinking and to mediate my expectations regarding the focus of their thinking. Here some of the students' comments showed more awareness of the educational purposes behind the work, and deeper understanding of the ways these goals can be achieved. In addition to this, there is evidence in many of the educational principles they had delineated of the influence our discussions had on their thinking.

The Video Lesson

The extent of the progress, however, became more clear during the second lesson of the third module. In that lesson, in which I showed videos of problem-solving lessons, many of the students began to express beliefs and attitudes which reflected the discussions we had had during the year.

> At first I gave them feedback on their assignments . (We talked about) the question of defining those goals - for instance, there were those that wrote their goal in doing sorting with the kids is that they should know what sorting is. I got them to give me some of the reasons that we want kids to sort - and they came up with quite a number of good ones. This has been one of my strategies this year - to get them to figure out what they should be looking for rather than tell them what it is. This is the first lesson where I've felt that I've actually managed to do that quite effectively. Thinking about it now, it could well be because this is not the first time they have thought about those things. Everything we talked about today was somewhat of a review of things already discussed. Which makes me think that now, after they have already gained some experience and have already thought about many important principles, the going might be much easier. I possibly will feel more comfortable now about giving up the reins of the lessons, and allowing them to get to the important things I have to say (sic!) on their own.
>
> (Journal, 20/1/98, lines 3-17)

The following day I wrote:

> Two nice lessons today. In both of them we had good debates on important issues. Maybe I'm deluding myself, but I seem to have heard arguments in both classes that showed real consideration of important principles. Not so much of the old taken-for-granted stuff.
>
> (Journal. 21/1/98, lines 3-5)

When I refer to both classes here, I am referring to both Wednesday classes, only one of which was part of the research. However, the same may be said for the Tuesday class as well. The beliefs and understandings of the students were in no way monolithic, with many of the students still expressing doubt about the possibility of putting into practice many of the ideas that had been discussed. What had changed, however, is that it was usually the students, not myself, who responded to students who expressed skepticism or beliefs that differed from the norms that were being developed. In the discussion on sorting one student suggested limiting the sets to be sorted so there would not be too many possible ways of sorting. There was much opposition to that suggestion:

> Others talked in favour of the richness - I felt the effects of the many times I had discussed the problem of trying not to "mix kids up" by limiting the richness of the mathematical activities...Then Tzila raised the social aspect of the activity - that the kids are listening to each other, and hearing each other's ideas. Donna related again to the question of whether there was too

> much stimulation, intimating that the world is like that and kids always deal with that.
>
> (Journal, 21/1/98, lines 21-32)

This lesson, which I saw as a watershed lesson, will be discussed in greater depth in the following chapter.

The Third Questionnaire

Two weeks after the end of the second module I gave the students a third questionnaire (see Appendix D) in which I asked for feedback on the first semester. It was completed by 26 full-time students, 16 in one group and 10 in the second. The two questions which related to the students' learning until then were, unfortunately, phrased in such a way that answers to the two could be appropriate to either one of them: "What is the most important thing that you have learned since the beginning of the year?" and "Have you changed your opinion about different subjects since the beginning of the year, and if so, what was the most significant change?" Because of this similarity between the two questions, I will report on the responses of both of them together.

These questions offered the students the possibility of listing as many points as they chose. The percentages, therefore, refer to the percentage of students that brought up points relating to each of the particular categories. From the large range of different responses, four general categories could be identified, which represented the largest number of responses. 23% of the students related in some way to the fear of mathematics - "It is possible to teach without causing them to fear the subject", "I found out that mathematics is not something that needs to be feared", etc. 35% of the students mentioned the possibility of teaching mathematics in a wide variety of ways. Just under 20% mentioned the fact that children can think independently and should be allowed to do so, and 23% talked about basing teaching on the child's prior knowledge. It may be seen that the comments of the students reflect the constructivist spirit behind our lessons and the awareness that mathematics can be taught very differently from the way they were taught as children, using a variety of materials and approaches, which will help children feel more comfortable engaging with the subject.

The discussion reported above, and the responses to the questionnaire, would seem to indicate the formation among the students in both sections of the course of a normative understanding of the essence of mathematics education based on a constructivist view

of mathematics learning. They were beginning to use the language of constructivism, thereby possibly constructing its ideas for themselves. Mead's (1934) understanding of language may be reflected in this process.

> Symbolization constitutes objects not constituted before, objects which would not exist except for the context of social relationships wherein symbolization occurs. Language does not simply symbolize a situation or object which is already there in advance; it makes possible the existence or appearance of that situation or object, for it is a part of the mechanism whereby that situation or object is created. The social process relates the responses of one individual to the gestures of another, as the meanings of the latter, and is thus responsible for the rise and existence of new objects in the social situation, objects depending upon or constituted by these meanings.
>
> (Mead, 1934, p. 78)

This development was to affect the way in which I was to conceptualize and operationalize the rest of the course, in that it provided me with the confidence to loosen my hold on our lessons and allow them to take the lead in a growing number of situations in our classes.

On the other hand, from remarks made by a number of students in feedback given anonymously immediately at the end of the second module, some of the students felt that they still had not been able to figure out exactly what it was that I was trying to do with them.

> It's a nice idea, this different way of learning, learning that is more profound. But it is difficult to put this idea into practice because it is hard for us to change our schemas of learning that we have in our heads about the subject of mathematics. For years we have been learning the material and practising it by solving exercises in order to understand or to learn it by heart.

Another student wrote:

> The approach is not yet so clear to me, although I understand its importance.

And a third wrote:

> This is the place for you to introduce and clarify first of all what the goals are of the course, what we are doing here, what you mean to teach us and what you mean to offer us so that we can help the children. The lack of the express presentation of your goals in my opinion mars the orderly progress of the lessons.

This last comment was particularly surprising in light of the great effort I felt I had made to mediate the meaning of the course both during the first module and continuing

into the second. The feelings of this student, as well as the comments regarding their difficulty in understanding the point of the course, may have been to some extent a result of my attempts to have the students mediate their own meaning rather than coming out straight and telling them how I see things - although from my point of view I was still doing too much of just that. On the other hand it may have reflected the difficulty that people have in changing long-held, well-rooted beliefs.

My Own Work

During this module I seem to have made quite a bit of progress in regard to my efforts to develop curriculum activities which allowed more active participation on the part of the students. Insofar as these attempts were manifest in my lesson plans, when I could reflect calmly on my goals and on conclusions and hypotheses that I had arrived at from previous lessons, this was a strategy that was easy to put into practice. Effecting change in the nature of my interactions with the students, however, was a different matter. The problem of my talking too much necessitated the creation of further strategies to facilitate the change of long-standing habits of interaction. Help was also to be had from the students themselves. Regarding one of our discussions in which I had tried to play a more central part, I wrote:

> It's interesting in these discussions that they really don't want me to butt in. I have a tendency to try, but they often don't let me. Today I was more successful in my attempts to control myself...
>
> (Journal, 21/1/98, lines 38-40)

A similar conclusion may be arrived at regarding my use of MLE. When I planned ahead of time to have the students relate to the mediational opportunities in the number stations activity, I better mediated both the actual activity and the mediation. Regarding my success in mediating modes of mediation that arose spontaneously in the lessons, I was far less successful. Two possible reasons for this may have been the extent to which I was overloaded regarding the focus of my research - rather than trying to put one action strategy into practice, I now had two that were demanding my attention. During this module, as I looked at the constructivist nature of my teaching, MLE necessarily took a back seat. Another reason, as I suggested regarding the first module, was that I needed to allow more time to learn to use MLE in such a way that it would become part of my teaching repertoire, rather than a strategy that needed to be consciously considered in order for it to take place.

CHAPTER SEVEN

THE THIRD MODULE - PROBLEM-SOLVING

The third module focused mainly on problem solving. In this module I attempted to take better advantage of context in my teaching. My growing understanding of the importance of classroom norms in effecting change in students' beliefs regarding mathematics education lent this element of our lessons particular importance.

During the first two modules the social constructivist theoretical framework of my research had already raised my level of awareness regarding opportunities to situate students' learning. I began to value one component of my lessons with which previously I had sometimes felt uncomfortable - my students' stories from the field. Although I naturally appreciated the fact that students told these stories, I had always worried that they were taking up precious lesson time. Now my concern with situating the students' learning caused me to understand the importance of this aspect of our lessons, to accept it as an important, if not crucial, component, and to consciously take advantage of it. .

Analysis of the data from the first two modules led me to identify two additional components of my teaching, both connected with my attempts at mediating for my students, which contributed to its situated nature. One was the mediation of the MLE that I did in the lesson, and the other was the use of situations in our lessons to point out important pedagogical points. Also, similar to the connections between the subject matter and the major action strategy of the first two modules, there was a parallel between the subject matter of this module, children's mathematical problem solving based on real-life situations, and the necessity of situating my students' pedagogical learning in real every day classroom situations. In this module, I intended to take greater advantage of all those elements of situated learning.

There was an additional action strategy that I decided to incorporate at this point as well. By the time we embarked upon this module, which spanned the end of the first semester and most of the second, I felt that the students were beginning to both grasp the general idea of a constructivist approach to mathematics education and to use its vocabulary. In addition, they were beginning to analyze various situations in terms of many of the principles that had been discussed during the first part of the year. This development, building on my understanding of the importance of context and social

interaction in the learning process, along with the dissatisfaction I was feeling with my overly-dominant role in our lessons, led me to try and hand over the reins to the students to a greater extent than I had done until then. The difficulty I was experiencing in doing so, however, which came about partly as a result of habit and partly from my continuing feeling that there were still things that it was important for me to tell them, led me to devise a strategy whereby I would separate the time set aside for the students' contributions from the sections of the lessons in which I was to present material in a more lecture-like fashion.

In this module the students themselves solved problems and explained their solutions, viewed videos showing children solving problems, and discussed typical solution strategies of children at different levels. We then moved on to work on the grammatical structure of problems, first analyzing the structure of the questions and then looking for reasons behind the relative difficulty of different kinds. Finally we began to look at ways in which mathematical subject matter can be introduced through problem-solving situations, specifically dealing with the subjects of pattern and multiplication and division. The subject of the base-ten system, although not approached using a problem-solving format, was included at the end of the module because of its connection with multiplication and division.

Situating the Learning

The time-ordered event listing of the third module shows that there was both wider and more conscious use of strategies to situate the students' learning. I continued to take advantage of situations that came up within the context of the lessons. I allowed more time to tell stories from the field, both my own and those of the students, I made use of videotapes of problem-solving lessons to allow my students to get a better sense of these lessons, and had group discussions about lessons that were planned together and carried out by each student in her field placement. The discussions that we had after viewing the video-tapes, as mentioned earlier, seemed to represent a turning point in the level of the students' understanding, and will be discussed in greater depth.

Taking Advantage of the Classroom Context

The first lessons of the module dealt with the subject of autonomous problem-solving - when given a problem without being instructed in a particular solution procedure, children will come up with a wide variety of solution strategies for the same problem

(Carpenter & Fennema, 1992; Hiebert et al., 1997; Franke et al., 1998). I posed a number of problems to the students at their own level, and had them present their solution strategies to the rest of the class.

> My impression of the lesson is that it was different from the lessons we've had until now. I felt that there was a kind of awakening because the character of the lesson was different than previous ones in my opinion. When Rachel gave us the problem...the students started working energetically. I didn't see anybody idle. The opposite was the case - everybody tried to solve the problem, they consulted with each other, they cooperated and they tried to deal with the problems and the different solutions...They were all so concentrated and so excited about solving the problem, and also about presenting their solutions to the rest of the class.
>
> (Dorit's summary journal, 13/1/98)

Having the students participate in mathematical activity, the way in which constructivist theory has "traditionally" been put into practice in constructivist-inspired mathematics teacher education, is effective in two important ways: the students have the opportunity to experience the kind of mathematics learning that a social constructivist approach can offer children; and, simultaneously, they have the opportunity to observe the teacher educator's practice. This may be used as a role model for their own practice in the future.

In addition, the wide range of strategies which they presented in the lesson was living proof of the fact that different people think differently, and therefore naturally solve problems in a variety of ways.

> I talked quite a bit about the importance of having the strategies come from the kids, rather than teaching them. How teachers decide on the strategies that the kids should use, and then are surprised that they don't understand - someone said that they should be trying to understand the kids instead!...
>
> (Journal, 14/1/98, lines 44-47)

My attempts at taking advantage of the students' problem-solving in the lesson by mediating its significance for work with children, as well as previous discussions of constructivism, seem to have paid off . While the above could be interpreted as only one student's conclusion, the atmosphere in the class seemed to indicate general agreement and enthusiasm. This atmosphere was part of and important for the continued building of educational norms which were distinctly at odds with the traditional understanding of teacher-student interactions. It provided the backdrop against which we then began to analyze the different solution strategies that are generally manifested by young children, possibly making the analysis of these

strategies more relevant, interesting and important in their eyes.

The Video Lesson

In the following lesson I showed videotapes depicting problem-solving lessons with children. The progress that I felt had occurred in the students' thinking during this lesson has been mentioned above in the evaluation of the second module. I will now present some of the exchanges between the students and attempt to analyze the kind and extent of change that had occurred.

In terms of understanding children's thinking and what they are capable of doing, videos would seem to be a close second to observing live classroom situations. In terms of the students learning to teach in this manner, the videos, while clearly not sufficient, can be of help as well. The conduct of this kind of problem-solving session with children is complex, and demands of teachers new kinds of skills and understandings. When handled well, the advantages of these lessons are striking. For both those reasons, the observation of lessons that videos make possible, accompanied by appropriate mediation and discussion, can do much to clarify their intent, engender deeper understanding of their various components, and provide the student teacher with evidence of their usefulness.

One of the videos shown, *Cognitively Guided Instruction* (Carpenter, Fennema & Franke, 1992), showed problem-solving lessons as well as interviews with the teachers of these lessons. The statements made by these teachers in favour of having children do their own learning, and of the role of the teacher as a facilitator, echoed many of the discussions we had had previously. Couched in different language, and situated in a different context, the video had the potential to reinforce the students' developing understandings. I also showed a video of similar lessons in Israeli classrooms, contributing to the understanding that this was not only possible elsewhere, but was already happening in real Israeli classrooms as well.

One of the problem-solving strategies that children use in the video was trial and error. I was interested both in clarifying the meaning of trial and error in these kinds of situations, and in having them consider the value or otherwise of this strategy.

> Rachel: There are those who think that trial and error is not an acceptable problem-solving strategy. I'd like you to think about that for a minute.

Sharon: They're not guessing.

Ofra: They're not guessing. They're trying a few different ways, in my opinion.

Sharon: Right.

Ofra: They keep on trying and trying, and in spite of the fact that they are making mistakes, in the end they succeed.

Rachel: How is it that they can start with a guess and in the end they succeed?

Ofra: There's a process. It's a process.

Rachel: What's the process?

Ofra: The process is through experience.

Martha: By checking.

Rachel: Who said "checking"? What do you mean by that?

Martha: The child checks himself, he sees that -

Sharon: He keeps on checking himself.

Martha: But I...agree with her. That a child who is possibly having some difficulty...checks and changes, checks and changes.

Rachel: Exactly. What's happening is that I do something through trial and error - at the beginning I just throw out something - but that gives me data. Then I check whether it is close to what I want, is it reasonable at all, and I learn from my guesses, I learn what's OK about them and what's not.

Conversation among students - unclear

Enid: It's the same thing. We learn from our mistakes.

Rachel: Exactly. We learn from our mistakes. If you answered a whole page of computation exercises correctly, it may well be a sign that you didn't learn anything from doing it.

Dorit: But it can also ruin a child, I think. If he makes a mistake, if he really makes a mistake, later he can make the same mistake over and over again. It's frustrating.

Rachel: O.K. The question of frustration is extremely important...

(Transcript, 20/1./98)

The above discussion may shed light on a number of changes that have occurred since the beginning of the year both in my teaching and in their understanding. At first glance this excerpt would seem to show a successful example of constructivist-based

teaching. Rather than simply explaining the ways in which trial and error can lead to a logical solution to a problem, I had the students mediate their own meaning by working out for themselves the ways that this might happen. On further analysis, however, my description of the way some professionals may look at trial and error as a solution strategy was a stimulus to have them think critically and professionally - to take into consideration both the present instance and issues that we had discussed in the past and to use them to understand more deeply the meaning behind this strategy.

In regard to the students' understanding, it seems that their responses were influenced by quite a number of points that had been discussed during the year: the importance of thinking in mathematics classes, the difference between guessing and conjecturing, the fact that learning is a gradual process that takes time, the place of experience in the individual's developing thinking, the importance of autonomy in learning rather than using teacher-determined solution paths, the counter-productiveness of children's fear of a teacher's wrath or their peers' ridicule when they err and the positive attitude toward mistakes that the teacher must hold: "The child who is possibly having difficulty - checks and changes, checks and changes", rather than the solution being corrected by the teacher, or receiving an explanation of the "correct" way to solve the problem. The students' analyses, therefore, seem to have been influenced by many of the norms of a constructivist approach to teaching that had become more and more accepted during the course of the year.

At the end of the excerpt it may be noted how Enid spontaneously connected the trial and error strategy with the way people generally learn from their mistakes. In this way she pointed toward an opportunity to mediate transcendence which I quickly took up. It is unfortunate, however, that after Dorit's last comment, that making mistakes might "ruin a child", rather than allowing this discursive process to continue by permitting the students to answer and discuss her reservation, I took the reins into my own hands once again, thus doing a disservice to the subject under discussion. Dorit's comment shows the way in which she continued to be concerned with the problematics of mistakes. In my response to her, however, I did not continue this thread, but decided instead to relate to the question of frustration in general, an interesting and important issue that I always enjoyed talking about. This was unfortunate in that it both turned attention away from the issue at hand, and cut off what may have been a continuing discussion. Until this point, however, the fact that I allowed the conversation to be directed by them seems to have reaped positive results.

The major theme of the videos that were viewed was the importance of allowing children to work out their own solutions to problems, rather than being shown one way by the teacher. Here I returned to my original strategy for mediating meaning by asking them to consider why this is important. Again, my intention was to have them consider this question based on norms of teaching that had been developed during the year.

Rachel Why is it important that they be the children's strategies? Why is it important that it not come from me, but rather that the strategies should come from the children?

Rochelle: You guide them. You mediate different things for them. But you can't take them and put them in their...thoughts...it develops the...

Rachel: Right - and by allowing them to be in control, they're active.

Rochelle: They don't just sit passively and listen to someone else.

Miriam: I think that that way it's much more meaningful for them.

Rachel: Right. Because it's their work.

Einat: Because it comes from them and it... (unclear).

Rachel: Right. And what about what the teachers said about that it's the children who are the teachers?

...

Edna: Children listen more to other children than they do to the teacher. Things that children say speak to them more.

Rachel: Right

Sharon: Children have similar heads.

Gili: But it's not always like that.

Rachel: It's certainly not always like that . But it often is. Children understand what other children don't understand...

Edna: They're coming from the same place.

(Transcript, 20/1/98)

In this dialogue, although I take a too-central place in evaluating and reinforcing what they say, the students once again are expressing attitudes regarding both the role of the pupil and the role of the teacher which have become normative in the class - that knowledge needs to be built by the children themselves, that the role of the teacher is to guide and mediate, not to "tell", that children need to be active in order to learn, that children can learn from each other, and that children can often understand each other better than adults can.

In spite of the atmosphere of general agreement and understanding of this approach to teaching, the students were by no means monolithic in their responses, nor did they accept uncritically what they saw.

> Sherry: What, are they like that in first grade?
>
> Rose: It's not possible. In first grade they barely know how to do 6-1.
>
> Sherry: I don't know about that. My son already knows.
>
> Rose: Yes, but there are many children that don't know yet and it takes them years to solve arithmetic problems.
>
> ---
>
> Tammy: In my opinion, [to make this possible] you need mathematics teachers who are at a very high level.

Although there were a number of students who continued to express their inability to believe that work at this level was possible with such small children, the debate was carried out among the students and the evidence for its feasibility was provided by them. In these exchanges I was no longer the central figure, the authority whose arguments held greater weight because of my position. These lessons show evidence of an increasing symmetry of classroom actions as portrayed by Bauersfeld:

> Both teacher and students contribute to the classroom processes. It is a jointly emerging "reality" rather than a systematic proceeding produced or caused by independent subjects' actions.
>
> (Bauersfeld, 1988, p. 29)

The students here were filling the role that peers play in apprenticeship situations, where different points of view and different levels of understanding stimulate argumentation regarding the issues at hand.

> Miller (1987) refers to argumentation as the central form of social exchange that brings about shared thinking in a way that advances individuals' knowledge and perspective. The principal feature of argumentation that takes it beyond other forms of social interaction is the need to find a collective solution to an interindividual problem of coordination. Such discourse has a built-in capacity to release processes of collective learning.
>
> (Rogoff, 1990, p. 178)

I felt that showing the videos had been very effective:

> I think the videos made quite an impression on them - although I know there is so much information contained in them that it's not possible to get it all in one viewing. All the excerpts...together did a good job of emphasizing the

> importance of kids figuring things out for themselves - and in the questions that I asked them at the end that was one of the things that I asked them to relate to.
>
> (Journal, 20/1/98, lines 20-33)]

Thus, the situations depicted in the videos offered an opportunity for my students to vicariously experience authentic teaching-learning sequences , to put their growing understandings into practice by attempting to analyze them, and to engage in collective learning through social interaction and argumentation. They led to exchanges which show how the developing classroom norms had influenced their analysis of a situation which at an earlier date might have sounded very different. These norms, and the language in which they were couched, could serve as tools for them to look critically at and evaluate teaching/learning situations, and develop understandings which would become part of both the group's normative view of mathematics education and students' individual belief systems with which they will enter the classroom in the future.

The discussions that took place in the video lessons in both groups gave me the confidence to do what I had tried to do from the beginning of the year - to allow the students more autonomy in the conduct of lesson. I had felt, however, that as long as their original conceptions of mathematics went unquestioned, I was not able to do this - I felt keenly the responsibility of presenting a novel and research-based professional stance toward teaching mathematics (Buchmann, 1986). It was only with this lesson that I began to feel I could leave many of the arguments to them - their professional understanding was good enough at this point in the year that I felt they were able to take my part in presenting the case for a constructivist approach to mathematics education.

The question remains as to whether this change, which received such dramatic expression in this lesson, had only then begun to manifest itself, or whether it could have been noted in previous lessons as well. On examination of the evidence it would seem that in the successful handling of the work on games by the Wednesday group reported in the previous chapter, this may have been the case. With the Tuesday group, there were no similar signs. While it seems clear that the significant transformation that occurred came as a result of a continuing process in both groups, it is also possible that the very positive atmosphere and expressed norms of the problem-solving lesson previous to the video-showing may have played a role in the understanding with which both groups approached the discussion in this lesson.

Perhaps an earlier viewing of the videos might have prompted this level of understanding. However, the ease with which the students subsequently continued to converse about the video, and the fact that while they were impressed by what they saw, they were on the whole not surprised by it, would seem to indicate that the timing of the viewing of the videos, coming after a period of development in their thinking, was probably instrumental.

Yanna, in her summary of the lesson, wrote:

> The students really took an active part in the lesson today. They brought up interesting issues, important questions and relevant problems. Each one expressed her own opinion about what she had seen. There was good cooperation in the lesson, and most of the students really listened to the opinions of their friends.
>
> (Lesson Summary, 21/1/98)

Among other things, a problem-solving approach to mathematics education encourages cooperation and tolerance toward different ways of acting and thinking. From Yanna's description of the students' discussion, it seems that something of the atmosphere of the problem-solving lessons depicted in the videos may have influenced the discussion that ensued in our own lessons as well!

Stories from the field

Brown et al. (1989) emphasize the importance of story-telling or narratives in the enculturation of individuals into a culture:

> Within a culture, ideas are exchanged and modified and belief systems developed and appropriated through conversation and narratives, so these must be promoted, not inhibited...
>
> The role of narratives and conversations is perhaps more complex than might first appear. An intriguing role in learning is played by "legitimate peripheral participation", where people who are not taking part directly in a particular activity learn a great deal from their legitimate position on the periphery (Lave & Wenger, in preparation). It is a mistake to think that important discourse in learning is always direct and declarative. This peripheral participation is particularly important for people entering a culture. They need to observe how practitioners at various levels behave and talk to get a sense of how expertise is manifest in conversation and other activities.
>
> (Brown et al., 1989, p. 40)

The stories both I and my students told, and the conversations that centered around these

stories, provided the opportunity to better situate the learning that was taking place in our classes, and to provide the students with the opportunity to participate peripherally in solving the pedagogical problems posed through these stories.

In the following lessons we continued to look at the subject of problem-solving.

> Rachel: It is important that they know why they are solving the problems, yes? That they're not doing it only because the teacher told them to. They need to do something that is significant to them. When they know why they are doing it, they want to do it.
>
> Hilit: I want to give an example. I was at the kindergarten on Tu B'shvat [a spring festival] and the teacher wanted to have them taste nuts and dried fruits. She wanted to give each child three almonds. She gave the first child three almonds, then she gave the next child five, but asked her to return the extras.. She gave the third child two and asked him how many were missing. She did that with the whole class . Before they could eat the almonds they had to figure out the answer.
>
> (Transcript, 25/2/98, p. 6)

Hilit's testimony from the field was another opportunity for students to see the theory carried out in practice. Here was an Israeli teacher who made use of a meaningful context to support her children's mathematical problem-solving. At the same time, the story itself, coming as it did from the field and providing the students with the opportunity to vicariously experience the lesson, could contribute to their own images and beliefs regarding effective pedagogy.

There were a number of occasions where stories told were based on, or had the effect of eliciting, the students' own childhood experiences. For one of the students, Lena, the positive experience of her student teaching was compared with her negative experience as a child. Her student teaching placement was in a class where the teacher, who had studied with me a number of years earlier, was putting into practice a problem-centred approach to the teaching of mathematics.

> Lena: At Forman School, my teacher Lillian...seems to be putting into practice what she learned. She works with the children with cookies, and she also...has them divide the cookies between them. It's really fun for them to learn mathematics, I think. It's fun for me too to see it because I remember how I learned (in school) and I see how this approach - it's completely different.
>
> Tzila: But do the children really <u>learn</u> that way?
>
> Lena: They love, they really <u>love</u> that lesson.

Yanna: What, I'm going to bring cookies every lesson?

Lena: Through problems. To teach them addition and subtraction through problems and not through dry computation exercises. Like she says - through problems.

(Transcript, 25/3/98, pp. 9-10)

Both Tzila and Yanna express skepticism regarding the feasibility of the teacher's approach. Lena's testimony, coming from the field, however, would seem to have the potential to reinforce the view of mathematics learning that I was encouraging, and to encourage further reexamination of persistent traditional views

The value of such reports, particularly in Israel, would seem to be enormous. At the time of this research the prevailing traditional practices in mathematics classes in most Israeli schools, did not allow many students to do their student teaching in this kind of class. This presented a major stumbling block in any attempt to change the current reality. Often students expressed admiration for the new ideas they met in the course, but simultaneously expressed their doubt that such a thing is possible in Israel. An exchange between students while watching the video a few weeks earlier hints at one perceived difference between classrooms in Israel and, in this case, in the United States:

Yanna: It's so quiet in the class.

Donna: It isn't Israel.

Meital: They're all wearing uniforms!

(Yanna's report, 21/1/98, p 3)

The above examples provided a glimpse into the way research-based mathematics education had proved itself to be not only effective but also possible in the Israeli context. Because these came at a point in the year where individual students had themselves put into practice, often quite successfully, many of the suggested activities, thereby convincing them of their effectiveness both in terms of the children's learning and in terms of the students' own enjoyment in teaching them, these testimonies of its practicality as a general approach to be used by the classroom teacher had the potential to further strengthen the students' developing beliefs and understandings.

Stories of both my own and the students' personal experiences and difficulties were also effective. When we looked at using problem-solving to introduce division, we discussed the possibility of beginning the work by giving children problems in which there is a

remainder:

> Rachel: The truth is that it is not good to always give them numbers which divide exactly.
>
> Noam: But it is easier and more satisfying.
>
> Dana: Why?
>
> Noam: First of all they practice with situations where it does divide exactly.
>
> Rachel: That's what they usually say. I don't really agree with that because if you start from -
>
> Miri: For me whenever there's a remainder I'm sure that I've made a computation mistake.
>
> A few students: Me too. Me too.
>
> (Transcript, 25/3/98, p. 4)

Miri's short comment about her own feelings provided a common emotional context for a number of the students, and highlighted the importance of the matter. The issue of the students' negative previous experiences with mathematics is one that has often come up in my didactics classes. These instances seem to provide opportunities for them to evaluate their own past experiences and reach conclusions regarding possible alternative ways of teaching that may prevent their own pupils from having the same kinds of experiences that they underwent as children.

In another instance I wanted the students to understand the difference between two different types of division situations - measurement division (the number of equal groups in a whole), and partitive division (the number of objects in each group). After having them work out the difference between the two on their own, a number of students were still having difficulty distinguishing between the two. When one student expressed the difference incorrectly, I sympathized with the difficulty many of them were experiencing and legitimized the mistake teachers often make by telling them about how I used to make the same mistake:

> Rachel: For many years when I taught young children I didn't realize that there was a difference between them. Nobody had ever pointed it out to me. A child would be given a problem like "I had 15 cookies and I wanted to divide them up between 5 children". Yes, imagine that that was the problem. And then I would help the child to solve the problem by saying "How many fives are there in 15?". I would take a partitive situation, which is one kind of division, and I would explain the problem using measurement division. They are two completely different situations. The choice of solution strategies is connected with the difference between the two situations.
>
> (Transcript, 25/3/98, pp. 7-8)

The use of my own stories of mistakes I made and misconceptions I had as a beginning teacher, both brought alive the difficulties the students might be experiencing and legitimized them. It also reinforced the understanding that good teaching is something that needs to be learned. It is not simply a natural talent that one either has or doesn't. Possibly a sign of the effectiveness of the story, Hilit's report of the lesson reinforced my feeling that they had shown particular interest in the discussion:

> There seemed to be quite a lot of interest in the discussion. Going over the transcript, though, it wasn't clear to me how many of the students were actively participating in the discussion - but in her notes on the lesson, Hilit mentioned how the students seemed to agree with the analysis of the different kinds of strategies, and to think it important as well.
>
> (Journal, 25/3/98, lines 42-46)

Further on in my journal I wrote:

> Telling the story connects what we are talking about with the reality of a teacher in the classroom - the learning becomes more situated. These stories are what make everything else real.
>
> (Journal, 25/3/98, lines 51-57]

I received further feedback on this lesson the following week when I made a comment which was misunderstood by some of the students to refer to that lesson. One of the students exclaimed "But last week's lesson was wonderful!". Although this comment was not made specifically in relation to that particular activity, it is likely that it reflected on that activity as well.

Myself as a Role Model

The possibility in teacher education of using the teacher educator as a role model for his or her students is a golden opportunity to situate students' learning. On first glance it would seem that the extent to which the teacher educator practises what s/he preaches is the extent to which this is made possible. As has been demonstrated earlier, and will be shown below, however, there can also be great educational value in reflecting on unsuccessful teaching as well.

In the problem-solving lessons there were numerous opportunities to have students experience the kinds of teaching that I hoped they would use in their own teaching in the future: from demonstrating how all students' solution procedures are presumed to be

logical, to showing how the role of the teacher is to listen to children's explanations as opposed to explaining, to expecting and encouraging many different strategies, to emphasizing the importance of the procedure as opposed to that of the answer...

On one occasion I found myself in an awkward position when I was not able to follow the explanation of a student's solution procedure. Although at first I felt extremely awkward and uncomfortable, later in the lesson I suddenly realized the pedagogical value of the situation, both for myself and for my students.

> There was something that came up twice in the last two lessons, and that's the possibility of using uncomfortable situations for me as learning situations for both myself and for my students....
>
> When later on in the lesson I referred back to that incident, excusing myself and saying that that doesn't usually happen, I all of a sudden realized how this is a perfect example of what happens to many teachers in these kinds of situations, and how I can use this occurrence to bring up some important points - about the difficulty of the teacher's position when she has to connect with the thinking of a lot of different children, and about the important part that the other children have to play in trying to understand each other's thinking.
>
> (Journal, 4/3/98, lines 72-98)

Although neither in this instance, nor in the other instance referred to in the journal, could my behaviour at the time be seen as a role model, it did highlight the kinds of situations that can occur in any teacher's practice. The way in which I realized the educational value of these situations, however, was an example of the way in which reflection on negative events can help a teacher learn and grow. Equally important for my students' learning was the fact that I made sure to mediate the meaning of these encounters, thereby increasing the likelihood that my students would understand their significance.

Summary

From the above account it may be seen that the learning during this module was, at least to some extent, grounded in realities that were meaningful to the students. As much as possible I attempted to both utilize the educational context of our own lessons as well as to bring the school classroom into our college lessons through the use of stories and with the help of videotapes.

The use of cases - stories, vignettes, episodes - can present and transport bits of life

to the classroom while still retaining much of their personal and particular flavour and meaning. This seems to be a particularly useful and effective strategy. Shulman defines a case as:

> "A piece of controllable reality, more vivid and contextual than a textbook discussion yet more disciplined and manageable than observing or doing work in the world itself"
>
> (Shulman, 1992, p. xiv)

Theoretically, therefore, not only are stories a way of introducing interesting and meaningful real-life situations into a college-based course, but they also have the advantage of being more manageable and immediately relevant to the discussion of a particular issue. In the work I did in this module it would seem that I only partially took advantage of the control offered by their use. When the stories were my own, I clearly was able to choose when and how to take advantage of them. The students' stories, however, left that to chance. When they brought them up I attempted to relate to them and further mediate their significance for our work. However, my concern about completing the planned subject matter constantly competed with this awareness. As I wrote at the end of the year, while going over my data,

> This is such an important strategy but I still didn't use it enough, even during the third module - I was still always too concerned with time and didn't encourage them enough to tell more stories about the things they were doing in school. It's important not only in terms of situated learning, but also in motivating them to try out more activities in class...
>
> (Note on Journal of 4/11/97 - 14/8/98)

The use of videos has many of the same benefits as stories, with the additional advantage that it allows the students to actually observe the occurrences as they take place in the classroom. Because technical difficulties made it impossible to view them in our regular classroom, it was impossible to interlace them with other activities, and I needed to limit their viewing to one lesson. Their value, as reported above, was considerable, which would seem to indicate the desirability of integrating them more into the course in the future.

Constructivism in The Third Module

Letting Them Do It

The data from the third module is largely testament to my growing comfort devising lessons which teach pedagogy in a manner more in tune with my constructivist beliefs. The time-ordered event listing shows a long row of activities which demonstrate this.

At the beginning of the module, I first introduced them to the categorization of children's problem-solving strategies as set out by Carpenter (1999). In an attempt to have them be as active as possible, I had them figure out the different strategies as described by Steinberg (1989) in article written in Hebrew. Because the descriptions of the strategies are difficult to follow, I had them work in small groups, each group discussing different sections of the article. They then presented the strategies to the rest of the class in the form of short skits. Dorit wrote of the group she observed:

> The group was very concentrated on the task. The students cooperated well with each other, they explained to each other, and tried to help each other understand...within seconds they built a convincing presentation.
>
> (Dorit's summary journal, 13/1/98)

In the following discussion, when we considered reasons for the posited relative ease of dynamic problems (in which there is an actual occurrence that takes place in the story - she had 6 strawberries, she ate 3 of them) as opposed to static questions (in one bowl there were three strawberries and in the other 5 strawberries), Michaela said "Because when there is an action it is easier to imagine it". Donna said "It feels more concrete that way". And Miri said "It is much easier when they can act it out or imagine themselves doing it". Regarding the reasons for starting with story problems before abstract arithmetic computation, one student, Deena, said "They're authentic questions".

The students' responses here indicated a number of principles of teaching that are in character with a constructivist approach - the importance of activity, of concrete understanding, and of the relevance of the problems to children's day-to-day lives. These responses, again, began to give me the feeling that I could leave the work to the students with the expectation that the discussion would be both high-level and meaningful to the students.

In the lesson where I had the students work out for themselves the differences between measurement division and partitive division, I asked them to demonstrate how they would solve this problem: "There are 39 cookies and 13 children. How many cookies will each child get?" Yvonne said that she would solve it in the same way that she had solved a previous one.

> Rachel: Could you explain how?
>
> Yvonne: I would draw it.
>
> Rachel: And what would you draw? You would draw 39 cookies,...
>
> Yvonne: And I would draw 13 children
>
> Rachel: Ah, so it's already different. Good. And then what would you do? You have 13 children.
>
> Yvonne: I would draw lines connecting the children to the cookies.
>
> Rachel: Good. That's not what you did before.
>
> Yanna: That's not the same thing. That's a different way.
>
> ---
>
> Miri: But you said before that you can take 39 cookies and divide them into groups of 13 so that you would get 3 groups.
>
> Rachel: Now, the question is whether that - it will give you the answer, that's true. But does that answer the question the way I asked it?...What does that say? What does that three mean?
>
> Hilit: That you have three groups and each child gets one cookie (from each group).
>
> (Transcript, 25/3/98, pp. 6-7)

In the above exchange, rather than correct Yvonne's statement that she would have solved the partitive problem in the same way as the measurement problem, I allowed her to explain her solution strategy and thus recognize her mistake, just as I would do in problem-solving situations with children. In this way I may have allowed her and the other students who were listening to the exchange to construct their understanding of the difference through their own work. In my journal I wrote:

> I think their feeling that the distinction is important and relevant came from the connection between the solution strategies and the distinction of the different kinds of division. Had I just told them about the different kinds they would not have really felt the importance.
>
> (Journal, 25/3/98, lines 46-57)

The following week, while reviewing the work we had done on division, I had planned

to list the reasons that it is important to begin the work with problems which leave a remainder. Instead of lecturing, I asked them to figure out what they thought might be my reasons for considering that a better alternative than the traditional way. My journal entry, while analyzing the potential benefit of the challenge I gave them, shows my feeling of discomfort with the way I phrased the task.

> I was about to tell them why I thought it a good idea to start with division with remainders, when I stopped myself and asked them if they could tell me what I think. Tamar seemed to appreciate the idea behind that. It's good, I think, because whether my principles are theirs or not, they should have picked up enough of what I think by now to be able to use that to figure out what I think about subjects that have never been discussed before. It seems to me that that's a good exercise in coming to conclusions - and one that they'll be able to take advantage of later in their own teaching. It is also a good way for me to see how well they have thought about and understood what is going on in the class, and a way that I can get them actively thinking rather than passively listening.
>
> (Journal, 1/4/98, lines 4-13)

On the one hand, I was able to spontaneously revise my plan to lecture about the advantages and instead asked the students to work out the reasons for themselves, thus seeming to indicate a growing awareness of teaching strategies which engage students more actively in the construction of their own knowledge. As I reflected in my journal, this may have been an effective strategy. On the other hand, my request that they figure out what *my* reasons might be, as opposed to deciding for themselves what *their* reasons might be, may indicate that my confidence in the students' own understandings and their ability to put these understandings into practice, was not yet as secure as I seemed to feel after the video lesson. This may also have been an indication of my continued difficulty in giving up the reins, no matter how competent many of the students were proving themselves to be in our discussions.

Separating

The continuing difficulty that I was experiencing in my interactions with the students expressed itself in my failure to allow them to conduct discussions on their own, and in the frustration I often felt when they did not allow me to complete my presentation of a particular topic. I needed to think of a strategy that would help me arrive at some kind of balance which would allow me to feel more comfortable with my teaching. I began to develop a plan to separate two distinct modes of teaching - the presentation of material in lecture form, and discussion in which the students are the major contributors and my main function is to act as a facilitator, evaluate the students' interactions and

mediate whenever appropriate.

From the beginning I presented the idea to my students, telling them that I had been dissatisfied with what I was hearing in the tapes of our lessons:

> I didn't like what I heard. What I felt was that there are conflicting needs. One is that I am interested in having you actively build your own knowledge, and that you should realize that that is how people learn better... I don't want to stand here telling you everything just as I don't want you to try and pour knowledge into your children's heads. So I really want you to talk and to think. I want you to take responsibility for your own learning so that you will learn more. On the other hand, there are so many things that I want to tell you - things that I have learned as a result of my own experience and reading. What I feel is that so far I'm constantly trying to both listen to you and to tell you information - and the clash between them is extremely unpleasant. What I've decided is to try and separate between the two - that there will be part of the lesson that is yours, that is the discussion part, when all of you can say what you think, and when the discussion is preferably between you, rather than between me and you. And then there will be part of the lesson that is more like a lecture - then it will simply be my turn to talk, without having you contribute to that part of the lesson. If there are things that you need to have clarified, you can obviously ask, but if there are things that you want to relate to I want you to make a note of them and we can discuss them after I'm done.
>
> (Transcript, 25/2/98, lines 3-20)

Actually putting this action strategy into practice proved to be sometimes difficult and sometimes problematic, and it met with varying degrees of success. One way in which I attempted to ensure my use of the strategy was by making notes in my lesson plans regarding which sections should be dealt with in which way. From the very first I realized how the separation had to make sense in terms of the form and content of the lesson - otherwise it would be so artificial as to be useless.

> According to my new decision to separate between the times when the students are supposed to talk and those when I am supposed to, I planned my lesson today such that the discussion of the different types of questions and their relative difficulty, would be divided into a first part in which they would answer questions that I put to them, and a second part in which I would add my own thoughts to what they said. I now realize that that was not good planning... if I want there to be a real separation between their part of the discussion and my part, the two parts have to be dealing with different subjects so that it makes sense to separate...In theory the separation idea sounds like a good one, but in this kind of circumstance at least, it doesn't seem to be appropriate. I will still try it out in a lesson where there is more of a natural separation between the parts.
>
> (Journal, 25/2/98, lines 4-21)

The artificial nature of the strategy led me often to neglect its use, in spite of it being part of the original plan of the lesson.

> When we returned to the part of the previous lesson that we hadn't finished yet... I had originally planned to just let them do the talking. In the lesson on Tuesday I had told them that that that was what I was going to do. In the lesson earlier that morning, I also tried to do it but without telling them. By this lesson I more or less forgot about the idea - it still feels too artificial and it didn't seem to work well with the other groups. That doesn't mean that I want to give up on the idea - I'll have to keep on trying.
>
> (Journal, 27/5/98, lines 13-20)

At the end of one lesson in which I had succeeded quite well in allowing the conversation to carry on between them, the students both cooperated and showed interest in the subsequent lecture.

> I ended up with barely a half an hour to talk about symbols - I had planned it to take 40 minutes, but even that was tight. This is where I reminded them of my idea to separate the parts of the lesson and told them to take out their notebooks to write down any ideas they had. I said that they could ask clarifying questions, but not add their own ideas - that they will be able to do next time during the review - I'm afraid that that review is going to take up most of the lesson! In any case, it worked. I talked straight for half an hour, and managed to finish everything I wanted to. At the end of it I was dead, as were they. Deena immediately told me how wonderful the lesson had been, as did a few others as well. I really do think it was good - a good mix of me talking, and them being active.
>
> (Journal, 28/4/98, lines 58-68)

I felt similar success the following week, when the main part of the lesson had been work in small groups and the subsequent reporting of their conclusions, where my part in the lesson had been almost entirely that of mediator and moderator of the discussion. It was then my turn to lecture. Deborah, in her report, writes:

> Rachel starts teaching about the base-ten system. She says it is now her turn to talk, and later on she will leave time for their comments. The students listen in <u>complete silence</u> while Rachel is talking.
>
> (Student's report, 6/5/98, p. 4)

From the data it would seem that my separating strategy had met with moderate success. It continued, however, to feel too artificial, and never really became a strategy that I could use smoothly and naturally. There was, however, a strategy that grew out of the idea of separation that was to prove extremely logical, natural, and easy to use: the review at the beginning of each lesson.

The Review

One of the most practical as well as effective strategies that I developed over the course of the year was the use of summaries, or reviews, at the beginning of each lesson. Although ostensibly simple and obvious, it was a strategy that offered solutions to the two particularly vexing problems of my work - the problem of my conscious use of MLE, which I have discussed at length above, and the tension between my desire to listen to my students, letting the conversation flow, and my desire to carve a path in their thinking which would lead to the development of a particular way of thinking about and practicing the teaching of mathematics. While the use of reviews greatly alleviated these problems, it also contributed to my efforts to situate their learning. In terms of the parameters of MLE, it offered me the opportunity to mediate transcendence, to make connections between different instances of the same phenomenon.

During the first part of the year there were a number of times when I started off the lesson by referring to previous lessons. The catalyst for this was the writing of my journals. When reflecting on the previous lesson I would often feel the necessity of further clarifying ideas that had been discussed, of relating to various situations that had occurred, or of bringing up important points that had been omitted. When, in the third module, I began to consider the idea of separating the lecture parts of the lesson from the more active student-participation parts, I began to recognize the advantages that this review had to offer.

> There are a few problems with the way I run the lesson that keep repeating themselves - one is the necessity of going back and filling in important things that were left out for one reason or another. This now connects up with the sectioning off of the lesson, which I don't seem to be doing very successfully. I have all these great plans about when I'll talk and when they will, about keeping the lesson balanced between experiential stuff and discussion, and now possibly add to that a part of the lesson that I keep as a review of the past lesson. It seems that I need at least 3 hours for every lesson rather than just 90 minutes. In this instance, it might be a good idea to use the first part of the lesson for summarizing what was done in the previous lesson, and bringing up any important points that were left out. It's obviously good as a memory jogger - both for them and for me - and it might put some of the new things we do that day more into context. As far as actually doing what I plan, I think this has the most hope of anything I've thought of till now. It just makes sense, plus it's at the beginning of the lesson so I shouldn't forget (!). It also needn't always deal strictly with the last lesson - if there are other things hanging, I can deal with those then too.
>
> (Journal, 25/3/98, lines 117-134)

The following week I presented the idea to the class:

> I mentioned my idea of doing a review at the beginning of the lesson, and they liked that very much.
>
> (Journal 31/3/98, lines 7-10)

Already on the following day my perception of the character of the review began to change. Rather than use it as lecture time, I decided to use it to bring up important points in a more participatory fashion.

> When I started doing the review of what we had done the previous week - it wasn't really a summary, but an opportunity to bring up missed points. I was about to tell them why I thought it a good idea to start with division with remainders, when I stopped myself and asked them if they could tell me what I think. Tamar seemed to appreciate the idea behind that.
>
> (Journal, 1/4/98, lines 4-7)

After one lesson, where I had neglected to mediate the ways in which I had mediated, I began to see the review as offering the easiest opportunity to mediate my mediation.

> Although I had planned it into the lesson, I didn't refer specifically to the MLE that was happening - this time more a question of time than of not thinking about it. It might be appropriate to include that in the review next week - maybe that should even be the regular time to do it.
>
> (Journal, 28/4/98, lines 53-56)

My lesson plan of the following week shows how I was beginning to learn to utilize the review time:

> Talk about the connection between symbols and the base-ten system - position as a symbol - and emphasize more the idea of position
>
> Remind them that the way we did it is not the way they should do it with children
>
> Ask them what they really prefer - that I give them more time to play each game, or that they should have the opportunity to try out as many games as possible.
>
> Talk about the MLE criteria that were present in last week's lesson
>
> (Lesson plan, 5/5/98)

This plan, as can be clearly seen, was far too ambitious for a "review" of twenty minutes, and indeed needed to be curtailed, but it demonstrates the enormous usefulness of the review for a variety of purposes. This, along with the ease with which it could be put into practice, quickly made the review a standard feature of every lesson. At the end of the year I wrote in my journal:

> I have a feeling that the decision to start each lesson with a review of the previous one has possibly been the only decision I have made about the structure of my lessons which I have carried through consistently. For a number of reasons it is both good and natural - I guess that's the winning combination...it refreshes their memories and helps things find their place in their minds. It also allows me to go back to points that I felt were not dealt with enough or were not clear enough, or even that had not been finished completely in the previous lesson...those things that I didn't get to because of lack of time, rather than being completely forgotten about, are at least mentioned, if not worked on seriously.
>
> (Journal, 9/6/98, lines 15-22)

Probably the major reason for the success of the review was the ease with which it could be executed. Because it mostly occurred at the beginning of the lesson (there were instances when I decided to change the order), I could move right into it with a fresh mind, concentrated on the plan which had been decided upon in the quiet of my own home, without yet having to take into account the multiple considerations of the classroom situation. These considerations, which had often been the nemesis of other action strategies that I had attempted to put into practice, were usually generated in the course of the lesson, not at the very beginning, and therefore did not prevent me from carrying out the review more or less according to plan.

MLE in the Third Module

Regarding my use of MLE, my success in using it consciously and mediating the mediation was somewhat uneven. At the beginning of the module I was still having difficulty with it, but later on I began to be more conscious of it and to relate to it as well. This was done mostly as a result of either planning the discussion of the criteria into my lessons, or through MLE cues which I wrote into my lesson plans - after having finished planning the lessons I would go over my plan and mark in where and what kinds of MLE were appropriate for which subject. All in all, of the 18 lessons that I studied in the third module, I both consciously used MLE and talked about it in about half of them.. After one lesson I wrote:

> What's interesting is that recently I've been feeling that the MLE criteria have gotten into my bones, and I am relating to them more and more naturally and easily - just when at the end of the first semester I was beginning to despair of that happening, it seems to have happened with my hardly noticing. To a certain extent I feel that that is what's been happening with some of the students as well. Although that was not so evident from this lesson, in the past few lessons, when I have referred to different criteria I have felt a growing understanding on their part of what I am talking about.
>
> (Journal, 4/3/98, lines 195-201)

This comment seems to have been somewhat over-optimistic - there were additional lessons towards the end of the module where I didn't have MLE on my mind at all. Nonetheless, on the whole, it would seem that both mediating consciously and talking about it were beginning, at this point of the year, to come more naturally.

The use of MLE in my analyses of my lessons, on the other hand, had become quite routine, and added greatly to my understanding of my practice. An example of this related to a short homework assignment that I had given them spontaneously:

> This was definitely the right instance of when to give a short bit of homework. When I gave them the homework I was once again mediating intentionality and reciprocity by telling them how important it is to understand this point, I was also mediating the feeling of competence by telling them that although it is confusing at the beginning, if they take the necessary steps they will understand it quickly, and also mediating goal achieving behaviour.
>
> (Journal, 25/3/98, lines 63-67)

EVALUATION OF THE THIRD MODULE

My Own Work

By the end of the third module I was beginning to see progress in my work on all fronts. The data shows that I succeeded in situating the students' learning on quite a number of occasions, and in a variety of ways - through stories, through the use of videos, and through basing our discussions on actual work that the students had done. However, the pressure that I felt regarding the time at my disposal caused me to use students' stories less than was desirable. This was a problem that I was aware of, and which demanded a suitable strategy, but which was not addressed in the course of this research.

Possibly my greatest success lay in my attempts to allow my students to take greater responsibility in our lessons. My growing feeling of comfort with this seems to have resulted from two major factors - my developing ability to plan constructivist-based activities which supported the development of the students' own pedagogical understandings, and the greater understanding that they themselves were showing which increased my confidence in their ability to raise important pedagogical considerations both in our whole-class discussions and in the small-group work.

Additional success could be seen in the implementation of the lesson review at the beginning of each session. This simple strategy proved to have the potential of being

particularly useful for many different purposes. It allowed me to better situate the lessons by connecting past and present occurrences, to mediate the ways in which I had mediated in the previous lesson, to allow my students to increase their understanding by returning to ideas discussed previously, and more...

Regarding my use of MLE, I seem to have made progress mainly through devising situations that would remind me to think about it and to mediate the mediating that I did. Some progress was also made in the degree to which it felt natural to me, but here I felt that more time and work would be necessary before that could actually come to fruition.

My Students' Work

My students were taking a much more active part in the lessons in this module than they had in the previous ones. This development seems to have been due to the two factors that I mentioned above - first, I was allowing them to do so by developing instructional activities which replaced lectures. Second, by the beginning of the third module their growing understanding and ability to reflect on pedagogical issues enabled me to loosen my control and give them more responsibility in the conduct of the lessons.

This development may have encouraged the building of classroom pedagogical norms. We discussed educational issues, the videotapes and stories told by both the students and myself. The fact that I left more room for the students in these discussions provided them the opportunity to verbalize their thoughts on pedagogy as well as to listen to those of others. The norms thus developed within our classroom community may be seen to be largely in accordance with social contructivist views of mathematics education: understanding the importance of experiential learning, having children think for themselves and encouraging them to construct their own understandings and solve problems in their own ways, appreciating the learning value of mistakes, and understanding learning as a process that takes place over time.

In the context of pre-service teacher training, the development of classroom norms is most importantly a means to encourage teachers to put these principles into practice in their own individual classroom teaching in the future. For this reason I began to see the importance of examining individual students' beliefs. The following chapter will examine to what extent the norms have actually become part of individual students' expressed beliefs.

CHAPTER EIGHT

A FOCUS ON BELIEFS

From the outset of this action research project I sought ways of making my course on mathematics pedagogy more effective and influential on students' future teaching practice. I chose to do this through the use of three major teaching/action strategies: through the use of Mediated Learning Experience in my teaching, through the teaching of pedagogy in a constructivist manner, and through utilizing context and social interaction in the development of students' thinking. Both the theoretical base of the work and the work itself led to a gradual shifting of focus during the course of the research. I originally approached teaching the course with a view towards encouraging the development of pedagogical content knowledge in mathematics in a way that would be as meaningful and significant for the students as possible. Although the importance of beliefs was taken into consideration, this was done as a means through which students could better develop their pedagogical content knowledge. As the year continued, a second way in which I attempted to further my goal was through the social context of the course. At that point my attention turned to the ways in which the students' understandings were shaped by the classroom learning community which gradually came into being over the year. At the close of the action stage of the research I shifted my focus once again, this time looking at my students' beliefs. In order to evaluate the effectiveness of the research project, i.e., to conjecture as to the extent to which the course would influence my students' future teaching, it became necessary to focus directly on the beliefs of individuals at its conclusion.

The literature indicates both the influential nature of an individual's beliefs on professional practices, and their potential to aid students in putting into practice the innovatory educational ideas encountered in their professional preparation in the college. As suggested by Renzaglia et al. (1997), novice teachers "who have a firmly established core of beliefs and practices" (p. 361) have a greater chance of acting as agents for change in their classrooms.

Beliefs

Ernest (1989) uses the term beliefs in a comprehensive way:

> What is referred to here as 'beliefs' consists of the teacher's system of beliefs, conceptions, values and ideology also referred to elsewhere as the

> teacher's 'dispositions' (Kuhs & Ball, 1986) The argument is that such conceptions have a powerful impact on teaching through such processes as the selection of content and emphasis, styles of teaching and modes of learning. In addition to subject matter related beliefs, the teacher's principles of education and views of its overall goals are also important (Wilson, Shulman, & Richert, 1987).
>
> (Ernest, 1989, p. 20)

I have chosen to adopt this broad interpretation of the term 'beliefs' for two reasons. Many researchers (Thompson, 1984; Wubbles, 1992; Hill, 1997; Carter , 1990; Griffiths & Tann, 1992; Gudmundsdottir, 1990, Zeichner & Tabachnik, 1985) have used similar, often parallel, terms. Reasons for refraining from the use of the term range from its too general nature which may lead to a lack of clarity, to its faith-like connotations, to the negative and unscientific implications associated with it. Beliefs, in a positivist culture, are seen as subjective constructions, as opposed to 'objective' knowledge.

> As a global construct, belief does not lend itself easily to empirical investigation. Many see it so steeped in mystery that it can never be clearly defined or made a useful subject of research. For these reasons, it is often seen as the more proper concern of philosophy, or, in its more spiritual aspects, religion.
>
> (Pajares, 1992, p. 308)

The effects of attempts at articulating clear distinctions between constructs closely connected with beliefs, however, result both in an emphasis on the differences between different categories, and a blurring of the distinctions between members of the same category. Regarding the often perceived dichotomy between beliefs and knowledge, these distinctions may result in misunderstandings such as the distinction between "subjective" and "objective" reality that led generations of positivists to claim the value-free nature and ascendancy of " fact" over subjective perception and understanding which largely determined the teaching strategies deemed appropriate for each.

Equally problematic is the blurring effect of this categorization. In trying to distinguish between beliefs and knowledge, one necessarily lumps different kinds and gradations of beliefs into one category. It is useful to look at the knowledge component of beliefs and use that to distinguish between different kinds, or different levels, of beliefs. These can be rated on a scale from those which are based entirely on personal, unexamined and uncorroborated ways of looking at experience, and those which have been analyzed and reflected upon, and therefore modified in accordance with one's growing knowledge of an educational domain. Beliefs can be characterized as more or less

grounded, depending on the examination they undergo and their potential modification.

The second reason for my choosing to use the word "beliefs" and for defining the term broadly, is connected with its powerful connotations. As opposed to alternative terms such as dispositions (Kuhs & Ball, 1986 in Earnest, 1988), theories (Griffiths & Tann, 1992), or perspectives (Zeichner & Tabachnik, 1985), the use of the word "beliefs" can be an indication of their importance in the conceptual development and change of the individual.

One of the connotations of the term "beliefs" is that of faith so strong that it cannot be questioned. Abelson (1979) identifies this as the existential presumption feature of beliefs. The strength of feeling associated with beliefs, and its resultant ability to direct action, has been recognized and accepted by educational researchers of all spectra - those who embrace beliefs and willingly incorporate them in their educational theories, and those who see them as undesirable elements of human thought which must nonetheless be addressed in educational work if that work is to be effective . Nespor (1987), having described the spurious process by which beliefs are formed, admits their strength of influence.

> And yet, for all their idiosyncrasies, [Nespor] concluded that they are far more influential than knowledge in determining how individuals organize and define tasks and problems and are stronger predictors of behavior.
>
> (Pajares, 1992, p. 311)

At the other end of the spectrum, Tillema and Knol (1997) devised a program for pre-service teachers in which they offer strategies to validate their students' existing beliefs and utilize their power in an effort to effect conceptual change. Their view of educational beliefs is as follows:

> Beliefs about teaching are perspectives on the profession, deeply-rooted 'certainties' and professional assessments or points of view about good teaching which are utilized in the evaluation of newly-presented information in preparation for future action (Pintrich et al., 1993)
>
> (Tillema & Knol, 1997, pp. 31-3

Tillema (1997) points out the impact of work on beliefs on the development of knowledge:

> Influencing belief change has a far greater impact on learning than do direct modes of presentation or instruction. It can be contended that programs that do not explicitly deal with student teacher beliefs will lead to less learning (at least at a reflective, or deep processing level) and consequently will not

> affect subsequent teaching performance, i.e. the deployment of new concepts in actual teaching in practical settings. It would be appropriate to envision beliefs as guiding the knowledge construction of student teachers.
>
> (Tillema, 1997, p. 294)

Changing Beliefs

As noted earlier, much literature (Lortie, 1975; Lacey, 1977; Ball, 1988; Tabachnick & Zeichner, 1991; Zeichner & Gore, 1990) has pointed to various socializing influences that affect the beliefs and attitudes of both pre-service and in-service teachers. According to Pugach (1992), those studies which are pessimistic in their estimation of the possibility that these influences can be confronted and changed are studies done in accordance with a functionalist perspective.

> [This perspective] implies that "teacher as change agent" is, for all intents and purposes, an unattainable goal and that socialization is basically a process that sustains conservative educational practice.
>
> (Pugach, 1992, p. 135)

The severity and pessimism of the functionalist tradition is expressed by Zeichner & Tabachnick (1985):

> On the one hand, first year teachers are seen as prisoners of the past (either anticipatory socialization or pre-service training) and on the other hand they are seen as prisoners of the present (institutional pressures emanating from the workplace). Significantly, in neither case are beginning teachers viewed as making any substantial contributions to the quality or strength of their own induction into teaching.
>
> (Zeichner & Tabachnick, 1985, pp. 3-4)

The alternative view, according to which it is possible to examine the absorption of new teachers into the school system is an interpretive one. According to this view, the socialization of novice teachers, while greatly influenced by the factors mentioned above, is nevertheless an individual process, one which is affected by the personal characteristics of the students, their beliefs, attitudes and actions.

> The interplay of institutional norms and idiosyncratic patterns require interpretation, assignment of meaning and understanding of process, and therefore fall into the interpretive tradition (Lacey, 1977).
>
> (Pugach, 1992, p. 137)

When looking at the literature on beliefs one generally accepted contention is

that they are difficult to change. Pajares (1992) discusses the importance of the stage at which beliefs are developed:

> Nisbett & Ross (1980) suggested that all people are theorists about their social and natural world and that information encountered early is the raw material from which they create the inferences they make about themselves, their surroundings and their circumstances. A primacy effect is at work as these early inferences bias interpretations of subsequent and often contradictory information, so that personal theories are always insufficiently revised even in the face of contradictions this new information may hold. Early experiences strongly influence final judgments, which become theories (beliefs) highly resistant to change...
>
> Due to these phenomena, the earlier a belief is incorporated into the belief structure, the more difficult it is to alter, for these beliefs subsequently affect perception and strongly influence the processing of new information. It is for this reason that newly acquired beliefs are the most vulnerable. With time and use they become robust, and individuals hold on to beliefs based on incorrect or incomplete knowledge even after scientifically correct explanations are presented to them.
>
> (Pajares, 1992, p. 317)

While accepting this primacy theory, Goodman (1988) believes that it is possible to affect these early beliefs. In a study of pre-service teachers' professional perspectives. She found that

> students did not enter (this program) with a hardened set of professional opinions. Rather, their pre-professional images formed an "intuitive screen" through which they interpreted their professional education...Unlike glass, which is rigid and does not allow wind, rain and sound to enter, screens connote a separation from the outside world , but one that is more open to external stimuli. This metaphor is useful for portraying the dynamics involved as students developed their perspectives of teaching...
>
> (Goodman, 1988, p. 130)

A number of models for changing beliefs have been put forth. Stuart and Thurlow (2000) have had students articulate their current beliefs so they can become conscious of them and examine them. Only in this way will pre-service teachers be in a position to change the status quo.

> As pre-service teachers begin their careers, they will be in a position to break this cycle, but they will be incapable of doing so as long as beliefs of which they are not cognizant drive their classroom practices.
>
> (Stuart & Thurlow, 2000, p. 119)

Korthagen constructed a program based on the influence that reflective thought can have on future action.

> A major aspect of the programme is that *reflection is stressed even before students embark on their practical teaching*. The idea behind this is that teachers can be armed against socialization into established patterns of school practice. The student teacher must first gain some idea of who he or she is, of what he or she wants, and above all of the ways in which one can take responsibility for one's own learning.
>
> (Korthagen, 1988, p. 39)

A number of researchers (Prawat, 1992, Tillema, 1997, Tillema & Knol, 1997) have utilized conceptual change research (Posner et al., 1982) in their efforts to understand the ways in which beliefs may be changed. Although Posner et al. emphasize the fact that their research is focused on learning as a rational activity, and for the purposes of their inquiry motivational factors are not taken into account, they admit the importance of these factors.

> [Learning] is fundamentally learning to comprehend and accept ideas because they are seen as intelligible and rational. Learning is thus a kind of inquiry. The student must make judgments on the basis of available evidence. It does not, of course, follow that motivational or affective variables are unimportant to the learning process. The claim that learning is a rational activity is meant to focus attention on what learning is, not what learning depends on.
>
> (Posner et al., 1982, p. 212)

The fact that researchers of belief-change use a theory focused on knowledge is another indication of the intertwined relationship between the two - each is affected by the other. For this reason Prawat chooses to utilize Posner et al.'s theory of conceptual change in an attempt to bring about change in beliefs.

> Getting people to change beliefs, especially intuitively reasonable ones, is a difficult proposition. Recent research on the conceptual change process indicates several criteria must be met: First, individuals must be dissatisfied with their existing beliefs in some way; second, they must find the alternatives both intelligible and useful in extending their understanding to new situations; third, they must figure out some way to connect the new beliefs with their earlier conceptions (Posner et al. 1982)
>
> (Prawat, 1992, p. 357)

Brown et al. (1989) offer another model of belief change in their construct of cognitive apprenticeship as an alternative approach to schooling. As an example of this approach they take Schoenfeld's work in which he and his students built a mathematical belief system through both his and the children's intuitive responses. In this work,

> Schoenfeld is consistently careful to emphasize that all...strategies are illustrated in action, developed by the class, not declared by the teacher. In

> his classes, the belief system is instilled in the only way it can be, through practice in which the students actively take part.
>
> (Brown et al., 1989, p. 38)

The above are examples of ways in which reflection, conceptual change, and participation in a social context have been used to encourage belief change in students. I will now show how my own course attempted to do the same through focusing on the three major action strategies: the use of MLE, the implementation of a constructivist approach to the teaching of pedagogy, and the utilization of context to situate my students' learning. Although for practical reasons these were approached in my research in a largely linear fashion, one strategy following other, I will attempt to show how they together constitute a model for teacher education which has the potential to significantly affect the beliefs of pre-service teacher education students.

Outcomes

Because this research looked at a year-long course during the students' pre-service teacher education, it was beyond its scope to return a number of years later and observe the actual teaching of individual participants as full-fledged teachers in the field. Indication may be had of the influence of the course, however, by looking at the quality of the students' testimonies regarding their perceptions and beliefs vis a vis mathematics education at the end of the year. Because this course is only a tiny fraction of the multiple school experiences that they have had and will have in the future, changes seen at its close can be viewed only as a beginning of a new attitude that will hopefully continue to develop and acquire both form and substance in the course of their future teaching. The question cannot be whether I "succeeded" in "producing" teachers of mathematics education according to a particular preferred model, but rather to what extent the course led to the students' changed perception of the field, and to what extent it offered them tools to look critically at alternative models of practice. Ernest (1989) puts the question thus:

> It has been noted that student teachers' idealism is 'washed out' by socialisation in schools (Lacey, 1977). Can teachers be helped to develop beliefs about mathematics and its teaching which are realistic and robust enough to resist this? What leads to shifts in beliefs and attitudes? Is it involvement in new practices, as curriculum developers suggest (Ahmed, 1987), or reflection on beliefs and practice?
>
> (Ernest, 1989, p. 25)

The key here is the extent to which belief change occurred - will their experiences in

this course, and the reflections that accompanied them, be sufficiently potent to be able to withstand difficulties encountered in their work with children as beginning teachers, and the heavily socializing influence of colleagues, principals and parents who more often than not continue to view mathematics education in traditional ways? The question of the extent to which they will be able to actually practise the conclusions that this altered view of the field dictates is one that remains outside of the scope of this project.

Evidence regarding the extent to which individual students' beliefs paralleled social norms which had developed over the course of the year was taken from two types of data. The first includes two assignments received from students toward the latter part of the year and the other is the end-of-the-year feedback questionnaire (appendix E). The assignments may be seen to offer a situated view of their perceptions - students express themselves in a more contextually-based manner, indicating the external stimuli and the internal thinking processes that led them to arrive at particular conclusions. The questionnaire provided a distilled summary of their general attitude towards mathematics education at the end of the year. It asked the students to comment directly on changes they experienced during course of the year, and, when compared to previous questionnaires (Appendices B,C,D) sometimes detected changes that might have not been directly expressed by the students.

Belief Change in My Students

In my analysis of belief change I have looked at the questionnaires and other available data which provided evidence either of change, or lack of it, in individual students. This data was provided mainly by the relevant questions in the third and fourth questionnaires, in which I specifically asked the students what significant changes they felt had occurred in their thinking, as well as a comparison of earlier questionnaires with later ones, particularly the first with the last. The reflective reports written for their final assignments often provided me with validation, as did freely offered written feedback which either stood on its own or was included in the response to the article written by Yackel et al. (1990). The main reason that this information was not available for all of the students was either because not all filled out all of the questionnaires, or because answers given were not sufficient to make reasonable conclusions. Altogether I found sufficient data regarding 27 students. A general overview of the data shows that some degree of change was in evidence in all the students. There were two cases, Rochelle and Tammy, where they had entered the course at the beginning of the year with a

strong belief in a more open and flexible style of teaching in general, and a belief that mathematics, too, could be taught in a similar way. Tammy nevertheless reports having changed in that she

> developed a wider view of mathematics as an activity that develops thinking in different areas - not only in regard to numbers.
>
> Tammy

Rochelle reports that she

> didn't really change her views, but rather began to develop a new perspective.
>
> Rochelle

A third student, Tzila, who expressed her skepticism at many points during the year, nonetheless reported change in her thinking, while expressing the need for further clarification:

> As a child for whom the experience of mathematics was difficult for me, I find that this approach can solve many problems that I had in the past, but there are still many question left open such as "How much time should be spent on different components of the lessons?", What comes before what and why?", and "Where does the teacher's role begin and end?" I think that there has been huge progress made in understanding children in the area of mathematics and I hope that more teachers will adopt this approach.
>
> Tzila

The categories of change that emerged from an analysis of the students' responses in the questionnaires are: (1) a change in the degree of comfort that the students feel regarding mathematics and their ability to teach it; (2) a change in their view of mathematics as subject matter; (3) a change in their view of the role of the teacher and pupil; (4) a change in the teaching methods they consider most appropriate; and (5) an increased awareness of important teaching decisions that need to be made, and ability to analyze or be critical of their teaching. Because the first two of these seem to go hand in hand, often reported by the same students and often referred to together, I will report on them in the same section.

Comfort with mathematics and a changed view of the subject

Of the 27 students that were considered here, 10 (37%) reported change in the degree of comfort that they felt with the subject of mathematics and/or with its teaching and 10 (37%) , to a large extent the same respondents, reported on a changed view of what

mathematics is. The percentage of reports of early difficulty with the subject contrasts with the data from the pre-project questionnaire which indicated that only 15% of respondents did not like mathematics as a subject and 15% did not feel comfortable with the prospect of teaching mathematics in school. The results of these end-of-the year testimonies were more in keeping with my original intuitive estimate, based on personal experience and a reading of the literature, which assumed a much larger percentage of this population of female pre-service early childhood teachers that would feel uncomfortable with the subject and its teaching. It would seem that in spite of the option that I offered them of completing the pre-project questionnaire anonymously, quite a number of the students who felt a lack of confidence regarding the subject, were loathe to share these feelings with the woman who was to be their teacher the following year. All of the following responses related both to their prior fear of the subject and to either to their changed understanding of what it is or of the way it can be taught.

> (Today) mathematics is less threatening to me. I enjoy doing activities with children that are connected with mathematics.
> And another thing - my view of mathematics is deeper and more all-encompassing - I always remind myself how I felt when I was a child and how children feel today when they are being taught mathematics.
>
> Meital

> Certainly!! All of a sudden mathematics is something nice and fun, and there is no need to be afraid of it. I discovered that the way you play a game is also important.
>
> Hilit

> Yes. I understood the importance of the didactic process for the learning of mathematics. I stopped being afraid of mathematics.
>
> Meital

The questionnaire responses and assignments of two students in particular, Sophie and Rose, are evidence of radical change in their attitudes, and was accompanied by marked progress in their pedagogical content knowledge. In her answer to the question of whether changes have occurred in her thinking during the course, Sophie answers:

> Of course there have been changes. I am no longer afraid of teaching lessons in this subject, nor am I afraid of working (on mathematics) with children. Because I saw that I can help them even with the little knowledge of mathematics that I have.
>
> Sophie

The question arises as to whether Sophie's feeling that she is able to help children is

accurate or not. Her reflective report handed in as part of the final assignment indicates that there is good reason to think that it is:

> I worked with a small group of children as I believe should be done in mathematics. I related to them both as individuals and as a group, and the interaction between the children was active. I provided them with manipulatives so they could use them if they want.
>
> I think that the lesson as a whole was successful. At the beginning I read the problem and it took them time to write it, time that I didn't have. I should have arrived prepared so as not to waste time on writing the question.
>
> There was one girl who asked me where the second number is in the problem because she saw only one number and was probably taught to look for two numbers in order to build the number sentence. I told her to read the problem over again. Then she discovered the thing about the ears (2). I think that girl went through an interesting process in the lesson. She figured out her difficulty on her own, and I think that strengthened her.
>
> Sophie

Even though Sophie's knowledge of mathematics as a discipline may be as limited as she suggests, it seems, in accordance with her evaluation, that her knowledge of teaching the subject to young children is far from being insufficient. In her report she relates to the importance of working closely with individual children, of interaction between the children, and of the children being active. She attempts to analyze the reason for the girl's difficulty, allows her to discover her own mistake, and indicates awareness of the empowerment that this can effect in a student. From the report it is impossible to discern the extent to which this awareness was existent before the lesson, but it certainly was reinforced as a result of the experience. She looks critically at her own work, analyzing both her mistakes and her successes. The quality of Sophie's report indicates that her belief that she is capable of teaching mathematics to young children is based on real understanding and knowledge. It also may be an indication that this understanding and the resultant belief will be well-enough based to inform her work with children in the future.

And from Rose:

> I would like to say to you, as we approach the end of the course, that over the year you provided me with a healing experience regarding the idea of mathematics. I never managed very well in this area, and I guess that is due to the education ministry, the teaching methods and the teachers that I had. With you I learned during the year that it is possible to do it differently. There are additional ways to teach it. By using a bit of thought and a lot of good intentions it is possible to turn mathematics into a magical and wonderful world for everyone.
>
> Rose

This feedback from Rose shows an extreme change in attitude - from seeing mathematics as threatening to seeing it as a "magical and wonderful world for everyone". As with Sophie, this declared change is backed up by the work she did with children at the end of the year. Her reflection on a lesson on multiplication in which she gave the children the task of dividing stickers into arrays of the multiples of three and six, shows both her understanding of the mathematical material as well as her ability to listen to the children and allow the lesson to develop in an organic way based on the children's own understandings.

> The strategies which the children used to solve the task that I gave them were grouping and division of countable collections (in this case stickers). The children used the three strategies that we discussed in our class [at the college]:
>
> - simple multiplication
> - partitive division
> - measurement division
>
> The children worked using direct representation (using the stickers) of the multiples of 3 and 6, but they succeeded in learning a number of principles in addition to those that I had defined for myself:
>
> From each individual set of arrays (of multiples of 3 or of multiples of 6) the children succeeded in concluding the commutative law by themselves: when they changed the order of the multiplier and the multiplicand the number sentence looks different but the result is the same.
>
> By comparing the two sets of arrays the children learned that each number in the table can be represented in more than one way, for example, 30 can be represented as either 3x10 or 5x6.
>
> By comparing the two arrays the children learned about remainders, and about a certain regularity in the remainders...that for each multiple of three that divides equally by six there is one that doesn't.
>
> By comparing the two arrays the children learned about the connection between multiples of three and multiples of six....
>
> The children learned many new and important things as a result of the cooperation between them and their group work. The reciprocal enrichment contributed to their arriving at important conclusions and guided the activity in directions that only this kind of brainstorming can do, even for young children (not only for adults).
>
> Rose

The knowledge of both mathematics and its teaching that Rose exhibits in this excerpt demonstrates the validity of her belief "that by using a bit of thought and a lot of good intentions" she is capable of teaching the subject in ways that are significantly different than those that she experienced as a child in school.

It may be conjectured that for Sophie and Rose, their significant pedagogical content knowledge may act as a solid base for their future teaching and for the further development of well-based beliefs regarding mathematics education.

Changed roles of teachers and pupils

19 of the 27 students (70%) specifically indicated change in their view of the role of the teacher and the pupil in the classroom. Of these there were four main categories: 6 of the 19 (32%) related to the need to base their teaching on children's prior knowledge and understanding (some of which specifically related to the teacher's responsibility to clarify what these understandings or misunderstandings are). Examples of these were:

> Children need to learn the way we learned this year - learning needs to happen through the children and through their ways of thinking.
>
> Elinor

> Not everything that is obvious to me is obvious to children. We must understand where the child has difficulties and help to solve them by coming from where the child is at.
>
> Karen

> We need to accept children's ideas and go along with them.
>
> Michaela

4 of the 19 (21%) refer to the ability of children to learn and solve problems on their own, and the importance of allowing them to do so. Gail wrote that at the beginning she thought that difficult problems would confuse the children and that they wouldn't be able to solve them independently.

> Today I know that problems can help them develop, and that they are capable of dealing with them.
>
> It's important to let children think and go through the process themselves until they reach a solution.
>
> Gail

Hagar felt the most important change for her was that she learned that it is possible to

> let children think independently and arrive at the solution themselves, not to teach them and limit them to only one solution strategy.
>
> Hagar

Anna's belief in children's ability to solve problems independently, which she

expresses in the final questionnaire, is backed up by her reflections in her final assignment, in which she had children solve word problems. Anna did not go along with my suggestion that all students solve the same single problem and subsequently explain their different solution strategies to each other, but rather decided to give each child a different problem to solve.

> In this activity I tried to develop the children's mathematical thinking and to show them that they were capable of solving the problems themselves and of explaining their solutions to others.
>
> I tried to get them to be attentive to each other by giving each one a different problem. In addition to the solution that each child gave for his or her own problem, I asked the others to think of different solution strategies for that problem.
>
> I provided the children problems, I provided them with concrete materials, and the rest they did themselves. In other words, children don't need the teacher to tell them if they're right or not, not only the teacher knows the answer. The children can arrive at the solution themselves, and moreover they can gain the self-confidence that they have the ability to do so.
>
> Anna

Anna suggests an explanation for her decision to give each child a different problem: that this will keep up the children's interest in the lesson. She tries to ensure their attention and active participation by asking them to think of alternative solutions to each other's problems. She emphasizes the children's potential to work independently, without using the teacher as an outside authority, and the benefited that can be gained by allowing them to do so. In the questionnaire at the end of the first semester Anna had already written that the most important change that had taken place in her thinking was the understanding that children can arrive at solutions on their own. This train of thought seems to have continued and developed in the course of the second semester as well, when she specifies more clearly her view that it is not her role to decide on what the correct strategy should be for individual children, and her understanding of the importance of independent problem-solving for the development of autonomy and self-confidence. The fact that Anna referred to this change in her thinking in both questionnaires suggests its importance to her, indicating the opportunity she had to develop and confirm this belief over time. Her belief may have been further reinforced through the problem-solving lesson she gave at the end of the year in which the children worked independently and in which she felt her role to have been restricted to providing the necessary conditions for learning to take place. This well-based belief may be a sign that Anna's future work with children will continue to reflect her learning during this course in mathematics education.

6 of the 19 (32%) refer to the importance of social interaction and the ability of children to learn from each other. Donna feels it is important to encourage interaction between the children "so they will think", Enid believes that children can learn from their peers, and Tzila

> likes group work because it gives more children a chance to take part. I like the cooperation, and that a variety of opinions can be expressed.
>
> Tzila

Shifra particularly noted the cooperation between the children in the box-sorting activity (see Burns & Tank, 1988) that she used for the final assignment.

> During the course of the activity I noticed the close cooperation that there was between the members of the group. I noticed that they tried to figure things out together and even explained their ways of sorting the boxes to each other. This activity provided each child with the opportunity to freely express their ways of thinking without any limitations, something which in my opinion gave them legitimacy and the desire to think. The children learned in an informal way to recognize the different ways of sorting - of thinking - and this time not from the adult figure but from the children themselves.
>
> During the activity there was a lot of thinking going on - the individual development of thinking while they sorted the boxes, and cooperative thinking while they were trying to guess the way the sorter had sorted.
>
> ... At the beginning I thought that the children would sort the boxes only according to size, colour and shape, I was surprised to see that their thinking was more open and varied, and of course more complex.
>
> I also noticed that the adult figure was superfluous, because the children did the activity without noticing me, the adult that was sitting with them.
>
> Shifra

Like Anna, Shifra felt that there was no need for her intervention during the activity. This is a radical departure from the traditionally viewed role of the teacher and indicates a strong understanding of the potential of well-planned activities to foster children's construction of their own knowledge. Nonetheless, while neither of them imply that this will always be the case, it must be noted that neither of them felt the need to mediate during the course of the activity.

Only one student, Hagar, specifically mentioned the importance of the teacher's mediation. Although it would seem significant that in the category of a changed view of the role of the teacher only one student did so, the fact that many of the students showed an understanding of MLE in their final assignments may somewhat offset a

conclusion that this was not an important change in their beliefs regarding the role of the teacher. For instance, Lois, in her response in the final questionnaire, wrote:

> The most significant change in my view of mathematics teaching is that you need to teach children mathematics by having them, themselves, solve problems, and only later teach them different techniques - in other words, to start from the prior knowledge of the child and not to teach according to one method.
>
> Lois

Although in this response Lois does not refer to mediation as being part of the role of the teacher, she does so in her final assignment in which she taught second grade children to play a trading game with base-ten blocks. She both incorporated the use of a number of criteria of MLE in her planning of the lesson, and referred to it in her summary:

> In this activity I saw the extent to which children can think on their own, and I saw the great importance of the mediation that I used in the lesson.
>
> Lois

Different and varied teaching methods

18 of the 27 (67%) students related to the issue of teaching methods. These included 2 responses which referred to the fact that it is important to teach children in the same way that they were taught in our course, since one of the most obvious novelties of the course was the many and diverse methods that were used. 12 of these 18 responses (66%) relate to the changed attitude towards mathematics that these diverse activities engender. Some of the students' responses were:

> For me mathematics is no longer just dry problems but also games and creativity. Learning math is more experiential and challenging. (It reduces the fear of mathematics.)
>
> Bella

> I thought there was only one correct solution and it was therefore necessary to teach it in a dry and monotonous way. Today I can teach mathematics in a versatile way.
>
> Gail

> There are different and diverse teaching strategies. This is a more fun and more interesting way to teach.
>
> Orianne

> It's possible to teach mathematics in such a way that it will be an experience for the children - interesting and varied.
>
> Rona

6 of the 18 responses (33%) relate to their belief that in this way children can learn more and better. Karen's response connects the use of diverse materials with the needs of individual children:

> Every child has a different way of thinking, so each needs to be taught differently - there needs to be diversity in teaching methods.
>
> Karen

In the final questionnaire Veronica testifies that:

> Today I am conscious of a variety of ways to teach mathematics in an experiential way. Because in my opinion only by working experientially using games and the like, will children like math and learn the subject in the best way possible.
>
> Veronica

Veronica's activity for her final assignment used geoboards, wooden or plastic boards with 25 nails arranged in a grid of 5 X 5 on which are placed elastic bands to form geometric figures. In discussing her reasons for carrying out this activity she wrote:

> First of all, in my opinion, this material varies the way in which geometry is taught. That's the reason it is important to bring it to class - in that way it turns geometry into something interesting and more enjoyable...
>
> I exposed the children to a variety of activities and materials, so that they would be aware of the fact that there aren't only books and worksheets in geometry, but there are also different games through which they can learn the subject.
>
> Veronica

The understanding of the importance of the use of a variety of activities and materials for Veronica is reinforced by the fact that she reported this as being the most significant change in the questionnaire at the end of the first semester as well. Although this fact may seem somewhat worrisome in that it could be seen to be of a more superficial quality than the other categories reported here, her reflective report on the lesson, which will be reported in the following section, shows good general understanding and an ability to think analytically about the children and critically about her own work.

For Rona, the importance of providing children with manipulatives was brought home to her in a problem-solving activity that she carried out with 7-8 year olds for her final assignment. When she asked the children how many vehicles of different kinds it would be possible to build from a certain number of wheels, she assumed that the children

would be able to imagine the wheels in their mind and work out their solutions abstractly. She quickly came to the conclusion that this was not the case:

> ...it's important to prepare wheels made out of paper so that it will be concrete for the child (something that I thought would be easy for the children to imagine and so I didn't prepare them at home, but on the spot I took a piece of paper and made them).
>
> Rona

Michaela, on the other hand, was already aware of the necessity of providing concrete materials. In her final assignment she reports on a game she had the children play where she provided manipulatives so they would be able to check their answers and know themselves whether they were right or wrong. She noted:

> They can check themselves (if they made a mistake or not) with the help of different concrete materials such as buttons, matches, etc. and by this self-checking they discover if they arrived at the right answer....
>
> I learned a huge amount from the children about their computational strategies. [I noticed] their hesitation about certain examples and their need to check and be sure.
>
> Michaela

Subsequent to the lesson Michaela may have begun to understand the importance of the use of manipulatives from the point of view of the child's confidence. From the above excerpt it is possible to speculate that this experience allowed her to begin to see the role that materials can have in promoting children's autonomy - with the help of the materials they are no longer dependent on the teacher to know whether their answers are correct, but can check themselves.

Analytical and Critical Approach

8 of the 27 responses (30%) indicated an increased awareness of important teaching decisions that need to be made, and the necessary analytical or critical stance that needs to be taken in order to make those decisions.

Tzila's report of change in the questionnaire shows her awareness of the importance of examining the degree to which activities are worthwhile as well as the ability to sit back and think critically about what she is doing:

> When I taught mathematics in the past I worked only from the curriculum - I

> did not come from where the children are. After our lessons I began to ask questions such as: What are we doing this for, What can the children get out of this?, Maybe they know more?
>
> Tzila

A number of the responses related to the students' awareness that they need to put aside their own perceptions of how to solve problems and allow the children to solve them on their own.

> In the future I will not decide ahead of time what way is right or wrong, but rather I will give the children space to think independently.
>
> Anna

Meira, also in the final questionnaire, wrote:

> you need to assume that all children's solutions and ideas are logical. The teacher and child together need to get to the logic behind it.

This statement may have been triggered by her experience in the final assignment, in which she had children solve a difficult multiplication word problem. In her report she records six different solution strategies used by the six different children who took part in the activity. To conclude, she writes:

> I did not reject any child's thinking strategy and I let them each work out the problem the way they chose - even when it happened that one child thought that he had done something wrong. I sat with him and we worked according to his own strategy and we succeeded in getting to the solution.
>
> Meira

Meira seems to have developed enough confidence in the basically logical character of children's thinking that it allowed her to be open enough to take the time to listen to and support the children in their solution strategies . Not only has she learned to listen well to the children, and to put her own ways of thinking aside while they are explaining their solutions, but she is also aware of what she is doing and seems to truly believe that it is both possible and desirable to act in this fashion.

In her report of a lesson given in geometry for the final assignment, Veronica brings up a number of points which show careful consideration and a good understanding of a number of important principles:

> -In this lesson I gave the children the opportunity to develop their thinking.
> -I exposed them to a variety of activities and materials so they would realize that there is more to geometry than just books and worksheets.
> -The activity encouraged social interaction. Children who completed copying the model helped those who were having some difficulty by giving them verbal instructions as to how to complete the task.

> -I encouraged verbalization - I asked each child who created a geometric design to describe it and the process of their work.
>
> Veronica

Veronica also shows a critical attitude towards taken-for-granted assumptions and an openness to discover unexpected abilities in children:

> For this activity I chose two children who are considered "strong" in the class and three who are considered "weak". It was interesting to see that the "weak" children enjoyed creating complex geometric figures (concave shapes), and preferred to use more than two elastics - so that it would be difficulty for the others to figure them out. The "strong" children, on the other hand, preferred to create simple shapes and used only two elastics.
>
> Veronica

Although the use of more elastics on the part of the two "weaker" children may in fact indicate a somewhat superficial understanding of complexity, the fact that the differences in the shapes created by the different children caused Veronica to question the accepted perceptions of the children's ability, may indicate a somewhat critical attitude on her part.. This impression is supported by her use of quotation marks for the words strong and weak, showing her tendency to question taken-for-granted precepts.

A number of students showed evidence of looking more critically at the material that they used than they had previously done. Liat writes:

> I see the games that children play in a more "mature" way - I see what they can learn from them.
>
> Liat

In the questionnaire Hagar shows evidence of looking analytically at her teaching also in reference to games by stating that "the <u>way</u> you use games is also important". There may be some doubt, however, about her ability to look critically at her own teaching. In the final assignment she gave a group of children in pre-kindergarten the problem of dividing thirty chocolates among 10 children. In her reflection she states that the activity was not appropriate for those children.

> I was very surprised by the children. There is no doubt that the problem was not appropriate for their level of development...I thought that they would have some notion about how to begin to solve it.
>
> One of the major problems in my opinion is that in this pre-school the children have almost no opportunity to do mathematics...
>
> I learned a lot from the lack of success of this activity. Work needs to be

> done on a regular basis with all the children, according to the age and level of each child. You can't expect an activity to go well without checking first what their level is. I am convinced that in my own class I will put the emphasis on different mathematical activities and will work regularly on them during the whole course of the year, just as we learned in the course and as I read in the article by Yackel.
>
> Hagar

Although Hagar's evaluation of the situation may have been correct, she refrains from looking at the way she carried out the activity. On the one hand she blames herself for not evaluating the knowledge before she began, but she does not analyze her own behaviour in the lesson or attempt to look for other possible reasons for its failure. From her reflective report it does seem nevertheless that the lesson reinforced her understanding of the responsibility of the teacher to know where the children are coming from, and she is determined in the future to both assume that responsibility and to offer her pupils the experiences necessary in order to encourage their mathematical development at the pre-school level.

Karen, at the end of the first semester, writes that the major change in her thinking was the realization that it is necessary "to take into account that not everything that is obvious to me is obvious to the child". This response would seem to indicate a critical approach toward decisions regarding what must be done if we are to succeed in helping children learn - we cannot simply act or explain in a way that might be appropriate for ourselves, but need to carefully look at the child. She continues,

> We must understand where the child has difficulties and help to solve them by coming from where the child is at...Every child has a different way of thinking so each needs to be taught differently. There needs to be diversity in teaching methods.
>
> Karen

Karen's response in the final questionnaire also referred to the necessity to "diversify teaching strategies in order to make as many children as possible feel comfortable with this subject". Although this response does not necessarily indicate a critical approach to mathematics education, it is interesting that in her report of the lesson that the students had planned cooperatively, Karen herself refers to the necessity of children looking at their own work in a critical manner:

> I think that asking the children questions is an important factor in causing the children to think critically about the operations they perform.
>
> Karen

In the lesson she gave for her final assignment of the year Karen, along with Elinor,

created a game requiring strategic thinking called "Rectangle Race". When they played the game with children they were able to analyze its successful and less successful features and subsequently modified it in accordance with their findings. This would seem to be another indication of Karen's awareness of the necessity to think critically as well as her ability to do so.

Summary

From an analysis of the data it would seem that quite a bit of progress occurred in the development of many of the students' thinking toward a social constructivist view of mathematics education. Only one student (Tzila) expressed reservations regarding the degree to which she understands and therefore fully accepts many of the claims of this approach. Two students, Rochelle and Tamar, held views of education in general that led them to expect a different approach to mathematics education. Rather than reporting change in their views, they felt that the year had provided them with an approach which was in keeping with their prior beliefs. All the other students for whom there was sufficient data, both report and show evidence of change in their beliefs regarding mathematics education.

CHAPTER NINE

REFLECTIONS ON MY LEARNING

At a distance from that intense and dizzying year of my research, I can look at my teaching and conclude that many of the hoped-for improvements in my practice did in fact take place. Since that time I have experienced a growing feeling of expertise, one which has allowed me to feel increasingly comfortable and self-confident as a teacher of teachers and as a professional mathematics educator. This occurred not only in those areas which were the explicit goals of my research, but also in my more general ability to interact with a class of students during the course of a lesson. Somekh's description of how action research changed her teaching practice echoes my experience as well:

> Action research provided me with a mechanism for managing change in my own classroom. It put me in control. It enabled me to use my intellect to understand the process of teaching and my own role in students' learning...
>
> Action research...provided me with a model of change which embodied principles of ownership and teacher professionalism, and was grounded in theories of individual and group behaviour derived from research (e.g. Lewin 1952; Schon 1983).
>
> (Somekh, 2000, p. 112-113)

At this stage in my development, as a result of my experiential learning and contributing to it, the literature on reflection has taken on new significance. In order to identify and understand the roots of this learning, therefore, I feel it necessary to take a look once again at the literature.

Reflection-in-Action

Eraut uses the term 'metaprocesses' to describe the thought processes involved in professional practice, distinguishing between those that occur in immediate interactive situations, and those that occur when there is time to reflect quietly on these interactions.

> Controlling one's behaviour involves the evaluation of what one is doing and thinking, the continuing redefinition of priorities , and the crucial adjustment of cognitive frameworks and assumptions... During rapid interaction self-direction is necessarily intuitive, drawing on previous experience with little deliberation. But when there is time for deliberation, it involves overall control of one's thinking, the informal scheduling of the deliberation, its conceptualization as a problem and as a process, and ongoing evaluation of its progress.
>
> (Eraut, 1994, p. 115)

This description would seem to recall Schon's distinction between two kinds of practitioner reflection: reflection-in-action and reflection-on-action. Reflection-in-action refers to practitioners' thought processes that take place within the framework of present action, while reflection-on-action refers to those that happen at a distance from that action. Although, intuitively, differentiating between these processes in terms of the time framework described by Eraut seems reasonable and obvious, Schon's meaning is different. He describes reflection-in-action thus:

> A practitioner's reflection-in-action may not be very rapid. It is bounded by the "action-present," the zone of time in which action can still make a difference to the situation. The action-present may stretch over minutes, hours, days, or even weeks or months, depending on the pace of activity and the situational boundaries that are characteristic of that practice.
>
> (Schon, 1983, p. 62)

Schon's first and primary goal is the differentiation between what he calls technical rationality - the cold and logical attempt to understand professional practice based on academic theory, and the practitioner's attempt to understand his or her own practice based on first-hand, complex real-life experience. He uses the term "swampy lowlands" to distinguish between the problems addressed by practitioners who choose to confront the problems of real-life situations and those who concern themselves only with problems addressed by rational, research-based considerations.

> There are those who choose the swampy lowlands. They deliberately involve themselves in messy but crucially important problems and, when asked to describe their methods of inquiry, they speak of experience, trial and error, intuition and muddling through.
>
> (Schon, 1983, p. 43)

The teaching instances Schon considers in the development of his theory are all examples of experienced professionals coaching novices on a one-to-one basis. He examines neither the situation of the classroom teacher faced with thirty children nor that of the teacher educator educating whole classes of future teachers. Nonetheless, possibly partly because of the evocativeness of the term "swampy lowlands", Schon's theory has drawn the attention of many educators concerned with classroom teaching. There would seem to be no better metaphor than that of a swampy lowland when considering the interactive teaching of a class. The term seems eminently appropriate both for the time-frame of interactive teaching and for the problems encountered within it.

Nor does Schon consider the case of classroom teachers attempting to improve their practice.

> ...in spite of the intuitive appeal of Schon's work to standard school settings and the obvious descriptive applicability of his view to classroom practice, it is not entirely clear how a teacher improves practice by reflecting-in-action. More centrally, how does a teacher improve reflection-in-action? The immediacy, pace and complexity of classroom life with thirty to thirty-five students is such that a teacher is in a constant state of acting and reacting. Given the press of a classroom, if a teacher's concepts (metaphors, images, understandings, constructs, etc.) for reflecting in action are narrow or inadequate, where is there a chance for their consideration?
>
> (Kilbourne, 1988, p. 93)

The main difficulty for teachers in the way Schon conceptualizes the notion of reflection-in-action may be the fact that it includes both the interactive moment of practice as well as the quiet moments in between when it becomes possible to engage in less urgent reflective thought. His main criterion for reflection-in-action is its potential to affect ongoing action, whether that happens in the space of a few minutes or a few months. Thus, relatively quiet reflection in moments of calm would often seem to more closely resemble reflection-on-action (Eraut, 1995). Eraut tries to find a solution to the problem by suggesting that if one looks at the word "in" in reflection-in-action as indicating context rather than time-frame, it would not be necessary to see reflection-in-action and reflection-on-action as opposites.

> The preposition *on* should refer to the focus of the reflection while the preposition *in* refers to the *context* of reflection...Reflection-in-action and reflection-on-action are not dichotomous opposites, indeed reflection-in-action is usually (though not always) on the action as well...O'Hanlon (1992) argues that it is reflection on action and for learning linked to future action that characterizes action research, not reflection-in-action or on-the-spot experiment.
>
> (Eraut, 1995, p. 16)

While this would seem a potentially fruitful solution to the confusion caused by Schon's choice of terms, it does not address the problem as I experienced it. While I agree with O'Hanlon that it is reflection on action that can affect future action, for myself the holy grail of my teaching was to develop the ability to better reflect while in the midst of my interactive teaching. My own experience of reflection-in-action had included numerous insights gained by the systematic writing of my journal, which I had the privilege to experience as a result of my action research, as well as sporadic insights arrived at in the course of my interactive teaching. I needed to learn to more consistently reflect-in-action in my interactive teaching. What was lacking for me in Schon's description of practitioner reflection, therefore, was the distinction that needed to be made between the

true swampy lowlands of interactive classroom teaching, and the quiet(er) situation of journal writing at the end of the day.

Van Manen (1991) provides an alternative restructuring of the reflection process which addresses the issue of time and more closely coincides with my own needs and perceptions.

Mindfulness

Van Manen delineates four forms of reflection which educators carry out in their educational role: *anticipatory reflection* which allows us to consciously and deliberately plan educational scenarios; *active or interactive reflection*, equated with reflection-in-action, which allows us to consider contingencies of a particular situation or problem with which we are confronted; *recollective reflection,* which allows us to come to deeper understanding of our past experience, and, finally, *mindfulness*, which, like reflection-in-action, relates to the interactive pedagogical moment, but is "a different type of reflectivity":

> In the immediacy of our actions, reflection does not occur in moments of interrupted stop-and-think action, neither does it occur parallel with our action. In other words, instant action is not usually produced by reflection. Yet this interactive experience or 'rush' itself may be mindful.
>
> (Van Manen, 1991, p. 513)

Van Manen holds that this type of reflectivity is structurally different from other kinds of reflection.

> The experience of thoughtful pedagogical action in pedagogical situations has a peculiar structure. It is neither largely habitual nor problem solving, neither intellectual nor corporeal, neither purely reflective in a deliberative sense nor simply spontaneous or arbitrary. Thoughtful action differs from reflective action in that it is thinkingly attentive to what it does without reflectively distancing itself from the situation by considering or experimenting with possible alternatives and consequences of the action. Living the pedagogical moment is a total personal response or thoughtful action in a particular situation. So when we come to tactful action, rather than say that it is 'reflective' we should say that tactful action is 'thoughtful' in the sense of 'mindful'.
>
> (ibid., p. 515)

A further description illustrates his concept more fully.

> No matter how well I have planned my lesson, or how enthusiastic I am

> about the subject matter, the interactive situation in the classroom is such that I must constantly remain aware of how it is for the students...and yet this awareness is more a thoughtfulness than a calculating or deliberative reflectiveness, which would put one equally out of touch with the students, since that would create a distance that accompanies any manipulative interpersonal relation between teacher and students. So as I interact with the students I must maintain an authentic presence and personal relationship for them...life in classrooms is contingent, every moment is situation-specific.
>
> (ibid., p. 518)

Van Manen's construct of mindfulness coincides with my own experience in that it sees the thought processes involved in interactive teaching and the reflection of more distanced journal writing as two separate and different forms of reflection, each with its own demands. Also, my experience of my teaching after the conclusion of my classroom research, so different from at its inception, coincides with the picture that he portrays. Assuming that these changes did indeed come about as a result of my action research, the question remains as to how this may have actually occurred.

Approaching Mindfulness

I would like to hypothesize two paths that my learning may have taken which eventually led to more mindful teaching. The first is that of the explicit learning that I achieved through my conscious reflection in the course of my self study action research. Eraut , following Kolb (1984) refers to this as experiential learning, learning which

> ...involves deriving explicit knowledge through reflection on experiences which might otherwise remain in episodic memory and be used only tacitly.
>
> (Eraut, 1998, p. 12)

This explicit learning would seem to have largely revolved around the questions with which I originally approached my research, as well as those that I began to focus on during its course. The other is the path of the implicit learning that I seem to have achieved during this same period and immediately following it. Evidence of this learning lies in the fact that at the end of the period of time during which I conducted the research my teaching seemed to have improved in ways and to a degree that could not be accounted for by the subjects that I had studied. Reber articulates one of the classic elements of implicit learning:

> Implicitly acquired knowledge is responsible for performance that goes beyond, as it were, what estimates of conscious knowledge would predict.
>
> (Reber, 1993, p. 40)

The explicit learning that took place, which has been discussed in the body of the thesis, will first be briefly summarized. The implicit learning that seems to have taken place, of which I became cognizant only at the conclusion of my research, will then be touched on.

My Explicit Learning

The explicit learning that came about as a result of this project may be divided into two categories: practical learning that has helped and continues to help me develop and improve my teaching, and theoretical learning which I find important for the understanding of the process of teacher education, and which also affects my teaching, but on a more general level.

Schon's account of problem setting would seem to be an appropriate opening to a discussion my explicit, practical learning. Problem setting, according to Schon, is

> ...the process by which we define the decisions to be made, the ends to be achieved, the means which may be chosen. In real-world practice problems do not present themselves to the practitioner as givens. They must be constructed from the materials of problematic situations which are puzzling, troubling and uncertain. In order to convert a problematic situation to a problem, a practitioner must do a certain kind of work. He must make sense of an uncertain situation that initially makes no sense.
>
> (Schon, 1983, p. 40)

In my quest to find ways to improve my practice I identified uncomfortable and vexing situations in my work, attempted to define the factors which caused them, and chose the means which logically seemed to provide potential solutions. This process was consciously contrived, carried out, and brought to some kind of conclusion. Conscious reflection on the lessons that I taught during the course of the year seems to have led to greater knowing-in-action (Schon, 1987, p. 25) regarding those specific issues that I had identified as objects of my research.

> When we set the problem, we select what we will treat as the "things" of the situation, we set the boundaries of our attention to it, and we impose upon it a coherence which allows us to say what is wrong and in what directions the situation needs to be changed. Problem setting is a process in which, interactively, we *name* the things to which we will attend and *frame* the context in which we will attend to them.
>
> (Schon, 1983, p. 40)

When I began this research project I had been teaching at the teachers college for about three years. In addition to the successes of those first few years, and the satisfaction that accompanied them, I experienced difficulties in my teaching which I felt quite at a loss to solve. Although I was doing my best to teach in accordance with the educational principles in which I strongly believed, there were many instances in which I felt I did not succeed. The main reason for this was, I felt, the difficulty of teaching early childhood subject matter in a way that could be challenging and involving for adults. I had envisioned and made many attempts to put into practice one particular solution strategy: devising activities for my students which were parallel in their content, and in the difficulties they posed for adults, to activities appropriate for young children. From my present point of view I can see that my feelings of frustration at the time stemmed, at least in part, from the lack of tools at my disposal which could assist me in devising additional, possibly easier and more effective solutions to this problem.

My experience conducting action research has introduced me to ways of thinking and acting which enable me to deal more effectively with the challenges posed by the character of the subject matter which I teach, and to identify and begin to solve additional problems of my teaching. I have become aware of ways in which I experience myself as a living contradiction - holding certain values and yet acting in ways that contradict them. One major advantage of this understanding is the realization that I no longer need to wish my teaching problems away - I now have the tools to confront them and deal with them in an intelligent and effective way. The result has been an improved fit between the values I hold to be important and those that I actually live by in my practice, as well as a strong feeling of empowerment in my work and in my personal life as well.

There are two particularly useful tools that I acquired as a result of my research. The first is the use of the reflective journal as a problem-solving tool. Systematic reflection on my teaching during my research enabled me to be constantly mindful of the needs of my students and of the classroom situation at any particular point in time, and provided me with the opportunity to consider problems which arose and to imagine and plan possible solution strategies. Although the daily demands of my teaching today do not generally allow me to take the time to reflect in writing, the periodic use of journal writing in order to solve particular problems of practice makes their resolution possible and the attempt to do so reasonably convenient. Again, for myself, one major benefit of this has been the resultant feeling of control over my work, a feeling which affects my teaching on a daily basis.

The criteria of Mediated Learning Experience (Feuerstein & Feuerstein, 1991) have provided me with a second tool that allows me to learn from my teaching. I have found that the analysis of my teaching with the help of these criteria, whether from a more general point of view or in looking at particular situations, is extremely useful, both as an aid in judging the effectiveness of my teaching and as a guide in helping me to increase that effectiveness. In terms of convenience, the criteria are remarkable. Having learned their significance and having internalized the specific meaning of each parameter, I can now use them as a readily available litmus test of the effectiveness of my teaching.

On a theoretical level, I have developed an understanding of the importance of utilizing both existing theory and practical experience in the search for solutions to practical problems. Theory as it is generally understood in the traditional realms of educational research (psychology, sociology, philosophy) can be helpful within limits. But attempts at transporting this theory in one block to the problems of practical reality, as well as attempts at achieving consistency in the understanding and utilization of this theory, are not suitable for dealing with the practical problems of teaching. This is due partly to their lack of connection with any particular contextual reality, and partly because of the necessity of looking at complex reality from many different perspectives, something that any one theory cannot do

> ...an important thing to remember about theories is that they may show different aspects of the same Reality. Because there is no one 'best' theory, we may need several rather different -looking theories about the same phenomenon in order to account for it.
>
> (Claxton, 1984, p. 4)

Clinging to one theory may thus lead to areas of blindness which will prevent practitioners from taking into account important factors in their attempts to improve their practice. For a teacher researcher whose goal was to improve the quality, and thereby the effectiveness, of my teaching, in whatever way this could be possible, claims for the exclusivity and consistency of constructivist theory on the one hand, and situated learning theory on the other, (Lerman, 1994; Lave, 1988) could be counter-productive. These arguments, which saw the theories as conflicting and therefore incompatible in one theoretical framework, I felt, would lead to the exclusion of insights which could contribute in important ways to the improvement of my practice. Seeing the understandings that different theories engender as relating to different facets of a complex reality, leading to a more inclusive and integrative view of theory as it relates

to practice, allows the practitioner to take advantage of the important contributions of each rather than excluding one theory in favour of another.

> ...the scientist and the learner differ somewhat in their primary goal and in their *modus operandi*. The research scientist's main concern about this theory is: Is it true? His job is to seek or construct any situation, however artificial or bizarre, that will reveal the limits or the flaws of his theory. The learner's primary concern, on the other hand, is very much the *utility* of his theory: Does it work? Does it do the job it is supposed to do under the conditions in which it was designed to do it?
>
> (Claxton, 1984, p. 21)

This would seem to be consistent with Whitehead, who holds that practitioners, in attempting to improve their practice, are in need of a living form of theory in order to address questions of the type 'How do I improve this process of education here?' He holds that theory, when stated in the propositional form of 'if...then' masks the living, dialogical approach in which the practitioner researcher experiences the negation of her or his own values within her or his own practice.

Human beings and society are complex entities that cannot be accounted for by a single, consistent theory. When the aim is to improve practice, enlightenment must be sought from whatever sources are available. The attempt to see the world through one consistent theory may be tantamount to putting blinders on our eyes that will prevent us from seeing both important problems and their potential solutions. Fortunately, it seems that the difficulties I encountered in my attempts at the outset of my research to reconcile the theories of situated learning and radical constructivism did not allow me to close off avenues of thought that were relevant and important to my work.

My Implicit Learning

A year after I had concluded the constructive stage of my research, the mathematics department of the college was moved to temporary quarters where the physical attributes of the classrooms should have made successful teaching that much more difficult. One room in particular was very wide and shallow, causing me to feel that I needed to constantly move from one side of the room to the other in order to ensure that all students took part. I emerged from my first lesson in that classroom triumphant. In spite of the objectively difficult circumstances, I felt that I had control of the class as I had never had before. In the following lessons I continued to feel the same way. Problems of discipline that so often plague Israeli classrooms both in the earlier school

years and in higher education had often stymied me and led to great frustration. Now, seemingly miraculously, they had almost disappeared. I began to feel as I imagine a true professional must feel when plying his or her trade.

Although I feel that I may be able to describe some of the abilities I have acquired, that is not the case regarding the processes by which much of this learning came about. It is clear that my reflections during the research process had an enormous effect on my teaching. The quantum leap I made in my teaching ability, however, would be difficult to ascribe to that conscious learning. A process of implicit learning was clearly at play in this development. Reber defines implicit learning thus:

> Implicit learning is the acquisition of knowledge which takes place largely independently of conscious attempts to learn and largely in the absence of explicit knowledge about what was acquired.
>
> (Reber, 1993, p. 5)

Clinical psychological experiments done by Reber (1993) and Berry & Dienes (1993) show that subjects exposed to various artificially structured situations, without being informed that any structure existed and therefore without making any conscious attempt to discover that structure, were subsequently able to identify exemplars or predict future occurrences. These included experiments with artificial grammars and probability experiments as well as more life-like situations. These experiments show in a striking way the ability that human beings have to "utilize the structural relationships inherent in ...complex stimulus domains" (Reber, 1993, p. 45), simply by being immersed in these situations.

Reber argues for the primacy of this implicit learning. Although he holds that implicit and explicit learning go hand in hand, he sees implicit learning as being the natural, default position.

> Taken together, these experiments lend general support to the proposition that implicit learning functions by the induction of an underlying representation that mirrors the structure intrinsic to the environment. Such an induction process takes place naturally when one is simply attending in an unbiased manner to the patterns of variation in the environment or when one is provided with an orientation that is coordinate with these variations.
>
> (Reber, 1993 p. 61)

Reber also points to the advantage that implicit learning has over more conscious attempts in regard to the ease with which such learning can be transferred to other

domains.

> The induction routine that appears to be operating in situations such as these is necessarily one that results in an abstract representation. Moreover, it can be applicable to classifying novel instances and not specifically characterizable by raw compiling of experienced instances.
>
> This issue is one of considerable complexity. The point of the preceding argument is not that all memorial systems must be viewed as being founded on induced abstractions. The evidence of Brooks (1978) and others (cf. Medin, 1989; Smith & Medin, 1981) shows that memories are frequently based on instantiations, fairly uninterpreted representations of the stimulus inputs. The point is that when implicit acquisition processes are operating, the resulting memorial system is abstract.
>
> (Reber, 1993, p. 57)

Another attribute of implicit learning that is relevant here is its robustness:

> One of the more compelling discoveries about unconscious cognitive processes is that they tend to be more robust than explicit cognitive processes; they typically survive neurological and psychological insults that compromise conscious, explicit processes.
>
> (Ibid., p. 18)

It would seem that the multiple experiences which I underwent as part of my action research could not help but result in a substantial amount of implicit learning in a relatively short period of time. Eraut (1998) makes the connection between experience, memory and implicit learning. In his model the linkage of past memories with current experience

> ...is there because the effects can only be explained as resulting from the accumulated experiences of several episodes rather than that of a single event. But there is no conscious awareness of the memories of these episodes having been combined to form a tacit knowledge base which enables future action.
>
> (Eraut, 1998, p. 3)

In tandem with the explicit learning of my research, and made more significant because of the primacy aspects discussed by Reber , it would seem that implicit processes might well explain the great strides that I took in my teaching as a result of this research. From a situated point of view, my development was a product of the context in which I was living and working, largely tacit in nature.

> Development involves individual effort or tendencies as well as sociocultural context in which the individual is embedded.
>
> (Rogoff, 1990, . 28)

Today, I still experience myself as a living contradiction whose values continue to be negated in my practice . In general, however, when confronted with problems in my teaching I no longer feel helpless. In my interactive teaching I have achieved a level of knowing-in-action which makes possible a new kind of mindfulness. The confidence I have developed as a result of having learned to reflect in-action and on-action allows me to view difficulties as challenging, food for thought, and the seeds of future change. Having completed this dissertation I am now at the point where I am ready to take my knowledge-in-action further through the enactment of a new action research project. I have identified more contradictions and am beginning to envision paths to their solution. And so on...

Bibliography

Ableson, R. (1979) Difference between Belief Systems and Knowledge Systems. *Cognitive Science*, Vol. 3, pp. 355-366.

Altrichter , H., Posch, P. & Somekh, B. (1993) *Teachers Investigate their Work.* London; Routledge.

Anderson, G.L., Herr, K. & Nihlen, A.S. (1994) *Studying Your Own School :An educator's guide to qualitative practitioner research.* Thousand Oaks, Ca.: Corwin Press.

Atkinson, S. (1994) Rethinking the Principles and Practice of Action Research: The tensions for the teacher-research. *Educational Action Research*, Vol 2, No. 3, pp. 383-401.

Ball, D.L. (1988) Unlearning to Teach Mathematics. *For the Learning of Mathematics*, Vol. 8, No. 2, pp. 40-48.

Ball, D.L. (1990a) Breaking with Experience in Learning to Teach Mathematics. *For the Learning of Mathematics*, Vol. 10, No. 2, pp. 10-16.

Ball, D.L. (1990b) The Mathematical Understandings That Prospective Teachers Bring to Teacher Education. *The Elementary School Journal*, Vol. 90, No. 4, pp. 449-465.

Ball, D.L. & Bass, H. (2000) Making Believe: The collective construction of public mathematical knowledge in the elementary classroom. In Phillips, D.C. (Ed.), *Constructivism in Education: Opinions and second opinions on controversial issues* (99th National Society for the Study of Education Yearbook), pp. 19-40. Chicago, Ill.: NSSE.

Baratta-Lorton, M. (1995) *Mathematics Their Way.* Menlo Park, Ca.:Addison Wesley.

Barnes, H.L. (1991) Reconceptualizing the Knowledge Base for Teacher Education. In Pugach, M.C. et al., *Changing the Practice of Teacher Education: The Role of the knowledge base.* Wash., D.C.: American Association of Colleges for

Teacher Education.

Bauersfeld, H. (1988) Interaction, Construction, and Knowledge: Alternative Perspectives for mathematics education. In Cooney, T. & Grouws, D. (Eds.) *Effective Mathematics Teaching*, pp. 27-46. Reston, VA: NCTM & Erlbaum.

Ben-Hur, M. (2000) Learning and Transfer - A Tautology. In Costa, A. (Ed.) *Teaching for Intelligence*, pp. 39-58. Chicago: Skylight.

Berliner, D.C (August/September 1986) *In Pursuit of the Expert Pedagogue.* Educational Researcher, pp. 5-13.

Berliner, D.C. (1994) Expertise: The wonder of exemplary performance. In Mangieri, J.M. & Collins Block, C. (Eds.) *Creating Powerful Thinking in Teachers and Students,* Chapter 7. Fort Worth, Texas: Holt, Rinehart & Winston. <http://courses.Ed.asu.edu/berliner/readings/expertise.htm>

Berry, D.C. & Dienes, Z. (1993) *Implicit Learning: Theoretical and empirical issues.* Hillsdale, N.J.: Erlbaum.

Blumer, H. (1969) Symbolic Interactionism. Englewood Cliffs, N.J.: Prentice Hall.

Bransford, J.D., Brown, A.L. & Cocking, R.R. (1999) *How People Learn: Brain, mind, experience and school.* Washington, D.C.:National Academy Press.

Brooks, J.G. & Brooks, M.G. (1999) *In Search of Understanding: The Case for Constructivist Classrooms.* Alexandria, Va.: Association for Supervision and Curriculum Development.

Brown, J.S., Collins, A. & Duguid, P. (January-February, 1989) Situated Cognition and the Culture of Learning. *Educational Researcher*, pp. 32-42.

Buchmann, M. (1986) Role over Person: Morality and authenticity in teaching. *Teachers College Record*, Vol. 87, No. 4, pp. 529-543.

Buchmann, M. (1987) Teaching Knowledge: The lights that teachers live by. *Oxford Review of Education*, Vol. 13, No. 2, pp. 151-163.

Burns, M. & Tank, B. (1988) *A collection of Math Lessons: From grades one to three.* Sausalito, Ca.: Math Solution Publications.

Burton, L. (1992) Becoming a Teacher of Mathematics. *Cambridge Journal of Education,* Vol. 22, No. 3, pp. 377-386.

Calderhead, J. (1981) A Psychological Approach to Research on Teachers' Classroom Decision Making. *British Educational Research Journal,* Vol, 7, pp. 51-57.

Carpenter, T., Fennema, E. & Franke, M. (1992) *Cognitively Guided Instruction.* Videotape. Distributed by Wisconsin Center for Education Research.

Carpenter, T.P. & Fennema, E. (1992) Cognitively Guided Instruction: Building on the knowledge of students and teachers. *International Journal of Research in Education*, Vol. 17, pp. 457-470.

Carpenter, T.P. & Fennema, E., Franke, M.L., Empson, S.B. (1999) *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, N.H.; Heinemann.

Carr, W. & Kemmis, S. (1986) *Becoming Critical: Education, knowledge and action research.* London: Falmer.

Carson, T.R. (1997) Reflection and its Resistances: Teacher education as a living practice. In Carson, T.R. & Sumara,. D. (Eds.) *Action Research as a Living Practice*, pp. 77-91. New York: Peter Lang.

Carter, K. (1990) Teachers' Knowledge and Learning to Teach. In Houston, W.R. (Ed.) *Handbook of Research on Teacher Education*, pp. 291-310. New York: Macmillan.

Carter, K., Cushing, K. Sabers, D., Stein, P., & Berliner, D. (May-June, 1988) Expert-Novice Differences in Perceiving and Processing Visual Classroom Information. *Journal of Teacher Education*, Vol. 39, pp. 25-31.

Clark, C.M. & Peterson, P.L. (1986) Teachers' Thought Processes. In Wittrock, M.C. (Ed.) *Handbook of Research on Teaching* (3rd Ed.), pp. 255-286. N.Y. Macmillan.

Claxtion, G. (1984) *An Introduction to the Psychology of Growth and change in Everyday Life*, Chapter 1. Milton Keyes: Open University Press.

Cobb, P. (1989) Experiential, Cognitive and Anthropological Perspectives in Mathematics Education. *For the Learning of Mathematics*, Vol. 9, pp. 32-42.

Cobb, P. and Bauersfeld, H., (1995), Introduction: The Coordination of Psychological and Sociological Perspectives in Mathematics Education. In Cobb, P. And Bauersfeld, H. (Eds.) *The Emergence of Mathematics Meaning*, pp. 1-16. Hillsdale, N.J.: Erlbaum.

Cobb, P. & Bowers, J. (March, 1999) Cognitive and Situated Learning Perspectives in Theory and Practice. *Educational Researcher*, pp. 4-15.

Cobb, P., Wood, T., & Yackel, E. (1993), Discourse, Mathematical Thinking, and Classroom Practice. In Forman, E.A., Minick, N. & Addison Stone, C. (Eds.), *Contexts for Learning: Sociocultural Dynamics in Children's Development*, pp. 91-119. New York: Oxford University Press

Cobb, P. & Yackel, E., (1998), A Constructivist Perspective on the Culture of the Mathematics Classroom, in Sieger, F., Voigt, J. & Waschesio, V. (Eds.) *The Culture of the Mathematics Classroom*, pp. 158-190. Cambridge, U.K.: Cambridge University Press.

Cobb, P. & Yackel, E., (1996) Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research. *Educational Psychologist*, Vol. 31 No. 3/4, pp. 175-190.

Cobb, P. et al. (1997) Mathematizing and Symbolizing: The emergence of chains of signification in one first-grade classroom. In Kirschner, D. & Whitston, J. (Eds.) *Situated Cognition: social, semiotic and psychological perspectives*, pp. 151-233. Mahwah, N.J.: Erlbaum.

Coffey, A. & Atkinson S. (1996) *Making Sense of Qualitative Data*. Sage: London.

Cohen, L, Manion, L & Morrison, K. (2000) *Research Methods in Education* (5th Edition). London: Routledge/Falmer.

Cole, A.L. & Knowles, J.G. (1998), The Self-study of Teacher Education Practices and the Reform of Teacher Education. In Hamilton, M.L. (Ed.), *Reconceptualizing Teaching Practice: Self-study in teacher education.* London: Falmer Press.

Cole, M. (1990) Cognitive Development and Formal Schooling: The evidence from cross-cultural research. In Moll, L.M., (Ed.) *Vygotsky and Education: Instructional implications and applications of sociohistorical psychology*; pp. 89-111. Cambridge: University Press.

Collins, A., Brown, S.B. & Newman, S.E. (1989) Cognitive Apprenticeship: Teaching the crafts of reading, writing, and mathematics. In Resnick, L.B. (Ed.), *Knowing, Learning and Instruction: Essays in honor of Robert Glaser*, pp. 453-493. Hillsdale, N.J.: Erlbaum.

Confrey, J. (1990) What Constructivism Implies for Teaching. In Davis, R.B., Maher, C.A. & Noddings, N. (Eds.) *Constructivist Views on the Teaching and Learning of Mathematics,* pp. 107-122. Journal for Research in Mathematics Education, Monograph No. 4. Reston, Va.: NCTM.

Corey, S.M. (January, 1954) *Hoping?* Or Beginning to Know!. *Childhood Education*, pp. 208-211.

Curriculum 2000 (2002). <http://mathcenter-k6.haifa.ac.il/tal_2000_grades.htm.>

Dewey, J. (1963) *Experience and Education.* N.Y.: Macmillan.

Dewey, J. (1997/1910) *How We Think*. Mineola, N.Y.: Dover

Doyle, W. (1977) Learning the Classroom Environment: An ecological analysis. *Journal of Teacher Education*, Vol. 28, pp. 51-54.

Dreyfus, H.L. & Dreyfus, S.E. (1986) Why Skills Cannot Be Represented by Rules. In Sharkey, N.E. (Ed.) *Advances in Cognitive Science 1*, pp. 315-335. Chichester: Ellis Horwood.

Dunlap, J.C. & Scott Grabinger, R. (1996) Rich Environments for Active Learning in the Higher Education Classroom. In Wilson, B.G. (Ed.) *Constructing Learning Environments: Case studies in instructional design*, pp. 65-82. Englewood Cliffs, N.J.: Educational Technology Publications.

Elliott, J. (1991) *Action Research for Educational Change*. Buckingham: Open University Press.

Elliott, J. (1995) What is Good Action Research? : Some Criteria. *Action Researcher*, Vol. 2, January, pp. 10-11.

Ely, M., Vinz, R. Downing, M, & Anzul, M. (1997) *On Writing Qualitative Research: Living by words*. London: Falmer.

Eraut, M. (1994) Developing Professional Knowledge Within a Client-Centered Orientation. In Guskey, T.R. & Huberman, M. (Eds.) *New Paradigms and Practices in Professional Development.*, pp. 227-252. NY: Teachers College Press.

Eraut, M. (1995) Schon Shock: A case for reframing reflection-in-action? *Teachers and Teaching: Theory and practice*, Vol. 1 No. 1, pp. 9-22.

Eraut, M. (November, 1998) Non-Formal Learning, Implicit Learning and Tacit Knowledge. Unpublished paper. University of Sussex.

Ernest, P. (1989) The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A model. *Journal of Education for Teaching*, Vol. 15, No. 1, pp. 13-33.

Ernest, P. (1991) *The Philosophy of Mathematics Education*. London: Falmer.

Evers, C.W. (1999), From Foundations to Coherence in Educational Research. In Keeves, J.P. & Lakomski, G. (Eds.) *Issues in Educational Research*, pp. 264-279. New York: Pergamon.

Feuerstein, R., (1990), The Theory of Structural Cognitive Modifiability. In Presseisen, B. (Ed.) *Learning and Thinking Styles: Classroom interaction*. Washington D.C.: National Education Association.

Feuerstein, R. & Feuerstein, S. (1991) Mediated Learning Experience: A theoretical review. In Feuerstein R., Klein, P. & Tanenbaum, A. (Eds.) *Mediated Learning Experience (MLE): Theoretical, psychosocial and learning implications,* pp. 3-51. London: Freund.

Feuerstein, R., Feuerstein R. & Schur, Y. (1997) Process as Content in Education of Exceptional Children, In Kozulin, A. (Ed.) *The Ontogeny of Cognitive Modifiability:Applied aspects of Mediated Learning Experience and Instrumental Enrichment.* Jerusalem: ICELP.

Feuerstein, Rand, Ryders (1988), *Don't Accept Me As I Am*. New York: Pleanum.

Feuerstein, R. & Rand, Y. (1997) *Don't Accept Me As I Am*. Arlington Heights, Il.: Skylight.

Feuerstein, R. (1999) Lecture given at Mophet Institute, Tel Aviv.

Flavell, J.H. (1963) *The Developmental Psychology of Jean Piaget.* Princeton, N.J.: D. Van Nostrand Co.

Flavell, J.H. (1977) *Cognitive Development*, Englewood Cliffs, N.J.: Prentice-Hall.

Fleer, M, (1990) Scaffolding Conceptual Change in Early Childhood. *Research in Science Education,* Vol. 20, pp. 114-123.

Franke, M.L., Carpenter, T., Fennema, E., Ansell, E., Behrend, J. (1998) Understanding Teachers' Self-Sustaining Generative Change in the Context of Professional Development. *Teaching & Teacher Education*, Vol. 14, No. 1, pp. 67-80.

Freeman, D. (1998) *Doing Teacher Research: From inquiry to understanding*. Pacific Grove: Heinle & Heinle.

Geertz, C. (1973) *The Interpretation of Cultures: Selected essays*. N.Y.: Basic Books

Gelman, R. & Gallistel, C.R. (1978) *The Child's Understanding of Number*. Cambridge, Mass.: Harvard University Press.

Goodman, J. (1988) Constructing a Practical Philosophy of Teaching: A study of pre-service teachers' professional perspectives. *Teaching and Teacher Education*, Vol. 4, No. 2, pp. 121-137.

Griffiths, M. & Tann, S. (1992) Using Reflective Practice to Link Personal and Public Theories. *Journal of Education for Teaching*, Vol. 18, No. 1, pp. 69-84.

Gudmundsdottir, S. (1990) Values in Pedagogical Content Knowledge. *Journal of Teacher Education*, Vol. 41, No. 3, pp. 44-52.

Hamilton, M.L. & Pinnegar, S. (1998), Conclusion: The Value and Promise of Self-study. In Hamilton, M.L. (Ed.) *Reconceptualizing Teaching Practice: Self-study in teacher education*, pp. 235-246. London: Falmer Press.

Hargreaves, D.H. (1996) *Teaching as a Research-Based Profession: Possibilities and prospects.* Teacher Training Agency Annual Lecture, UK.

Hawkins, A. & Sharp, T. (1992) Field Systems Analysis: In search of the expert pedagogue. *The Journal of Teaching in Physical Education*, pp.

Hiebert, J. et al. (1997) *Making Sense: Teaching and learning mathematics with understanding.* Portsmouth, N.H.: Heinemann.

Hill, L. (1997) Just Tell Us the Rule: Learning to teach elementary mathematics. *Journal of Teacher Education*, Vol 48, No. 3, pp. 211-221.

Hitchcock, G. & Hughes, D. (1995) *Research and the Teacher: A qualitative introduction to school-based research* (2nd Ed.) London: Routledge.

Holt, J. (1964) *How Children Fail. New* York: Pitman.

Howe, K.R. & Berv, J. (2000) Constructing Constructivism, Epistemological and Pedagogical. In Phillips, D.C. (Ed.), *Constructivism in Education: Opinions and second opinions on controversial issues* (99th National Society for the Study of Education Yearbook), pp. 19-40. Chicago, Ill.: NSSE.

Hubermann, A.M. & Miles, M.B. (1998) Data Management and Analysis Methods. In Denzin, N.K. & Lincoln, Y.S. (eds.) *Collecting and Interpreting Qualitative Data*, pp. 179-210. Newbury Park, Ca: Sage,

Kamii, C. (1985) *Young Children Reinvent Arithmetic*. New York: Teachers College Press.

Kilbourne, B. (1988) Reflection on Vignettes of Teaching. In Grimmett, P. & Erickson, G. (Eds.) *Reflection in Teacher Education*, pp. 91-112. NY: Teachers College Press.

Killion, J.P. & Todnem, G.R. (1991) A Process for Personal Theory Building. *Educational Leadership*, March 1991, pp. 14-16.

Klein, M. (2001) Constructivist Practice, Pre-Service Teacher Education and Change: The limitations of appealing to hearts and minds. *Teachers and Teaching: Theory and practice*, Vol. 7, No. 3, pp. 257-270.

Kohl, H. (1969) *The Open Classroom: A practical guide to a new way of teaching*. New York: Vintage Books.

Korthagen, F.A.J. (1993) Two Modes of Reflection. *Teaching and Teacher Education*, Vol. 9 No. 3, pp. 317-326.

Korthagen, F.A.J. (1988) The Influence of Learning Orientations on the Development of Reflective Teaching. In Calderhead, J. (Ed.) Teachers' *Professional Learning*, pp. 35-50. London: Falmer Press.

Kozol, J. (1967) *Death at an Early Age*. New York: Bantam.

Kozulin, A. (1994) Cognitive Revolution in Learning: Piaget and Vygotsky. In Mangieri, J. & Collins, C. (Eds.) *Creating Powerful Thinking Teachers and Students*, pp. 269-287. Fort Worth, Texas: Harcourt Brace.

Kozulin. A. & Presseisen, B. (1995) Mediated Learning Experience and Psychological Tools: Vygotsky's and Feuerstein's perspectives in a study of student learning. *Educational Psychologist*. Vol. 30, No. 2, pp. 67-75.

Lacey, C. (1977) *The Socialization of Teachers*. London: Methuen.

Lather, P. (1991) *Getting Smart.: Feminist research and pedagogy within the postmodern*. N.Y.: Routledge.

Lave, J. (1988) *Cognition in Practice: Mind, mathematics and culture in everyday life*, pp. 3-34, Cambridge: Cambridge University Press.

Lave, J. (1993) The Practice of Learning. In Chaiklin, S. & Lave, J. (Eds,), *Understanding Practice: on Activity and Practice*, Cambridge, Cambridge University Press

Lave, J. (1996) Teaching, As Learning, in Practice. *Mind, Culture, and Activity*, Vol. 3, No. 3, pp. 149-164.

Lave, J. & Wenger, E. (1991) *Situated Learning: Legitimate peripheral participation*. N.Y.: Cambridge University Press.

Leinhardt, G. & Greeno, J.G. (1986) *The Cognitive Skill of Teaching*. Journal of Educational Psychology, Vol. 78, No. 2, pp. 75-95.

Leinhardt, G. (1988) Situated Knowledge and Expertise in Teaching. In Calderhead, J. (Ed.) *Teachers' Professional Learning*. London: Falmer.

Leinhardt et al (1991) Where Subject Knowledge Matters. In Brophy, J. (Ed.) *Advances in Research on Teaching*, pp. 87-113. Greenwich, Conn.: JAI Press.

Lerman, S. (1994) Articulating Theories of Mathematics Learning in Ernest, P. (Ed.), *Constructing Mathematical Knowledge: Epistemology and Mathematics Education*, pp. 41-49. London: Falmer Press.

Lindquist, M.M. (1997) Foreword. In Hiebert, J., Carpenter, T.P., Fennema, E. Fuson, K., Wearne, D., Murray, H., Olivier, A., Human, P., *Making Sense:Teaching and Learning Mathematics with Understanding*. Portsmouth, N.H.: Heinemann

Livingston, C. & Borko, H. (July-August 1989) Expert-Novice Differences in Teaching: A cognitive analysis and implications for teacher education. *Journal of Teacher Education*, pp. 36-42.

Lomax, P. (1995) Accounting for Ourselves: The problematic of representing action research. *Cambridge Journal of Education*, Vol. 25, No. 3, pp. 301-313.

Lortie, D.C. (1975) *Schoolteacher: A Sociological Study*. Chicago: The University of Chicago Press.

McNiff, J. (1988) *Action Research: Principles and Practices*. London: Routledge.

Mead, G.H. (1934) *Mind, Self & Society*. Chicago: University of Chicago Press.

Miles, M.B. & Huberman, A.M. (1994) *Qualitative Data Analysis: An expanded sourcebook (2nd Ed.)*. Newbury Park, Ca: Sage.

National Council of Teachers of Mathematics (1991), *Professional Standards for Teaching Mathematics*, Reston, Va.: NCTM.

National Council of Teachers of Mathematics (1989), *Curriculum & Evaluation Standards for School Mathematics*. Reston, Va.: NCTM.

McNiff, J. (1988) *Action Research: Principle and Practice*. London: McMillan.

Neill, A.S. (1960) *Summerhill*. New York: Hart Publishing.

Nesher, P. (1999) Television Report in *Uvda*. Interviewer: Yael Dayan.

Nespor, J. (1987) The Role of Beliefs in the Practice of Teaching. *Journal of Curriculum Studies*, Vol. 19 pp. 317-328.

Newman, J.M. (1999) Validity and Action Research: An online conversation. In Hughes, I. (Ed.) Action Research Electronic Reader online. <http://www.behs.cchs.usyd.edu.au/arrow/reader/>

Noddings, N. (1990) Constructivism in Mathematics Education. In Davis, R.B.,

Maher, C.A. & Noddings, N. (Eds.) *Constructivist Views on the Teaching and Learning of Mathematics*, pp. 7-18. Journal for Research in Mathematics Education, Monograph No. 4. Reston, Va.: NCTM.

The Nuffield Foundation (1971) *Nuffield Mathematics Project*. London: W&R Chambers and John Murray.

Pajares, M.F. (1992) Teachers' Beliefs and Educational Research: Cleaning up a messy construct. *Review of Educational Research*, Vol. 62, No, 3, pp. 307-332.

Peterson, P.P. & Comeaux, M.A. (1987) Teachers' Schemata for Classroom Events: The mental scaffolding of teachers' thinking during classroom instruction. *Teaching and Teacher Education*, Vol. 3, No. 4, pp. 319-331.

Phillips, D.C. (2000) An Opinionated Account of the Constructivist Landscape. In Phillips, D.C. (Ed.), *Constructivism in Education: Opinions and second opinions on controversial issues* (99th National Society for the Study of Education Yearbook), pp. 19-40. Chicago, Ill.: NSSE.

Piaget, J. (1971) *Psychology and Epistemology: Towards a Theory of Knowledge*. Kingsport, Tennessee: Penguin Books

Polanyi, M. (1966) *The Tacit Dimension.* Garden City, NY: Doubleday

Posner, G.J. et al. (1982) Accommodation of a Scientific Conception: Toward a theory of conceptual change. *Science Education*, Vol. 66, No., 2, pp. 211-227.

Prawat, R.S. (1992) Teachers' Beliefs about Teaching and Learning: A constructivist perspective. *American Journal of Education*, Vol. 100, No. 3, pp. 354-395.

Pugach, M.C., (1992), Uncharted Territory: Research on the socialization of special education teachers. *Teacher Education and Special Education*, Vol. 15, No. 2, pp. 133-147.

Reber, A.S. (1993) *Implicit Learning and Tacit Knowledge: An essay in the cognitive unconscious.* Oxford: Oxford University Press.

Renzaglia, A, Hutchins,M., & Lee, S. (Fall, 1997) The Impact of Teacher Education on the Beliefs, Attitudes and Dispositions of Preservice Special Educators. *Teacher Education and Special Education*, Vol. 20, No. 4, pp. 360-377.

Richardson, L. (1998) Writing: A Method of Inquiry. In Denzin, N.K. & Lincoln, Y.S. (Eds.) *Collecting and Interpreting Qualitative Materials*. Thousand oaks. Ca.: Sage

Rogoff (1990) *Apprenticeship in Thinking: Cognitive Development in Social Context*. N.Y.: Oxford University Press.

Rudduck, J. (1986) Teacher Research, Action Research, Teacher Inquiry: What's in a name? In Rudduck, J., Hopkins, D., Sanger, J., & Lincoln, P. (Eds.) *Collaborative Inquiry and Information Skills*. British Library Research Paper 16, Boston Spa: British Library.

Russell, T. (1988) From Pre-service Teacher Education to First Year of Teaching: A study of theory and practice. In Calderhead, J. (Ed.) *Teachers' Professional Learning*, pp. 13-34. London: Falmer.

Russell, T. & Munby, H. (1991) Reframing: The role of experience in developing teachers' professional knowledge. In Schon, D., *The Reflective Turn: Case studies in and on educational practice*, pp. 164-187. N.Y.: Teachers College Press.

Samaras, A.P. & Gismondi, S. (1998) Scaffolds in the Field: Vygotskian interpretation in a teacher education program. *Teaching and Teacher Education*, Vol. 14. No. 7, pp. 715- 733.

Schempp, P. et al. (1998) Differences in Novice and Competent Teachers' Knowledge. *Teachers and Teaching: Theory and practice*, Vol. 4, No. 1, pp. 9-20.

Schon, D. (1983) *The Reflective Practitioner: How professionals think in action*. N.Y.: Basic Books.

Schon, D. (1987) *Educating the Reflective Practitioner*. San Francisco: Jossey-Bass.

Schneider, W. & Fisk, A.D, (1983) Attentional Theory and Mechanisms for Skilled Performance. In Magill, R.A. (Ed.), Memory and Control of Action, pp. 119-143. N.Y.: North Holland Publishing Co.

Shulman, J.H. (1992) *Case Methods in Teacher Education* N.Y.: Teachers College Press.

Shulman, L.S. (1986a) Those Who Understand: Knowledge Growth in Teaching, in *Educational Researcher*, February, 1986.

Shulman, L.S. (1986b) Paradigms and Research Programs in the Study of Teaching: A contemporary perspective. In Wittrock, M.C. (Ed.) *Handbook of Research on Teaching* (3rd Ed.), pp. 3-34. N.Y.: Macmillan.

Shulman, L.S. (1987) Knowledge and Teaching: Foundations of the new reform. *Harvard Educational Review,* Vol. 57, pp. 1-21.

Shulman, L.S. (1992) Toward a Pedagogy of Cases. In Shulman, J.H., *Case Methods in Teacher Education* N.Y.: Teachers College Press.

Simon, M. (1995) Reconstructing Mathematics Pedagogy from a Constructivist Perspective. In *Journal for Research in Mathematics Education*, Vol. 26, No. 2, pp. 114-145.

Skemp, R.R. (1986) *The Psychology of Learning Mathematics* (2nd Ed.). London: Penguin.

Smith, J.K. (1995) The Ongoing Problem of Criteria. In Tiller, T., Sparks, A., Karhus, S. & Dowling Naess, F. (Eds.) *The Qualitative Challenge: Reflections on Educational Research,* pp. 133-154. Hagerupsvei, Norway: Caspar Forlag.

Solomon, J. (1994) The Rise and Fall of Constructivism. *Studies in Science Education*, Vol 23, pp. 1-19.

Somekh, B. (1983) Triangulation Methods in Action: A practical example. *Cambridge Journal of Education*, Vol 13, No. 2, pp. 31-36.

Somekh, B. (1995) The Contribution of Action Research to Development in Social Endeavours: a position paper on action research methodology. In *British Educational Research Journal,* Vol. 21 No. 3, pp. 339-355.

Somekh, B. (2000) Changing Conceptions of Action Research. In Altrichter, H. & Elliott, J. (Eds.) Images of Educational Change, pp. 111-122. Buckingham: Open University Press.

Stake, R. (1995) *The Art of Case Study Research*. Thousand Oaks, CA: Sage

Stenhouse, L. (1975) *An Introduction to Curriculum Research and Development*. London: Heinemann.

Stuart, C. & Thurlow, D. (2000) Making It Their Own: Preservice Teachers; Experiences, Beliefs, and Classroom practices. *Journal of Teacher Education*, Vol. 51, No. 2, pp. 113-121.

Tabachnick, B.R. & Zeichner, K. (1984) The Impact of the Student Teaching Experience on the Development of Teacher Perspectives. *Journal of Teacher Education,* Vol. 35, No. 6, pp. 28-36.

Tatto, M.T. (1998) The Influence of Teacher Education on Teachers' Beliefs About Purposes of Education, Roles and Practice. *Journal of Teacher Education,* Vol. 49, No. 1, pp. 66-77.

Thompson, A.G. (1984) The Relationship of Teachers' Conceptions of Mathematics Teaching to Instructional Practice. *Educational Studies in Mathematics*, Vol. 15 pp. 105-127.

Tillema, H.H. (1997), Reflective Dialogue in Teams: A vehicle to support belief change in student teachers. *European Journal of Teacher Education*, Vol. 20, No. 3, pp. 283-296.

Tillema, H.H. (2000) Belief Change Towards Self-directed Learning in Student Teachers: Immersion in practice or reflection on action. *Teaching and Teacher Education*, Vol 16, pp. 575-591.

Tillema, H.H. and Knol, W.E. (1997) Collaborative Planning by Teacher Educators to Promote Belief Change in their Students. *Teachers & Teaching: Theory and practice*, Vol. 3, No. 1, pp. 29-45.

Tiller, T, (1995), Action Learning and Action Research - Opportunities and Dilemmas. In Tiller, T., Sparkes. A. , Karhus, S. & Dowling Naess, F. (Eds.), *The Qualitative Challenge: Reflections on educational research*, pp. 37-54. Hagerupsvei, Norway: Caspar Forlag.

TIMMS, Third International Mathematics and Science Study (1995). <http://nces.Ed.gov/timss/results.asp>

TIMMS, Trends in International Mathematics and Science Study (1999). <http://nces.Ed.gov/timss/results.asp.>

Tzuriel, D. (1991) Cognitive Modifiability, Mediated Learning Experience and Affective-Motivational Processes: A transactional approach. In Feuerstein R., Klein, P. &Tanenbaum, A. (Eds.) *Mediated Learning Experience (MLE): Theoretical, psychosocial and learning implications*, (pp. 95-120). London: Freund.

Van Manen, M. (1991)Reflectivity and the Pedagogical Moment: The normativity of pedagogical thinking and acting. *Journal of Curriculum Studies*, Vol. 23 No. 6, pp. 507-536.

Voigt, J. (1989) Social Functions of Routines and Consequences for Subject Ma□tter Learning. *International Journal of Educational Research*, Vol. 13, pp. 647-656.

Voigt, J. (1992) Negotiation of Mathematical Meaning in Classroom Processes: Social interaction and learning mathematics. Paper presented at The International Conference for Mathematics Education, Quebec, Canada.

von Glasersfeld, E. (1987) Learning as a Constructive Activity. In Janvier, C. (Ed.) *Problems of Representation i n the Teaching and Learning of Mathematics*, pp. 3-18. Hillsdale, N.J.: Lawrence Elbaum Associates.

von Glasersfeld, E. (1990) An exposition of Constructivism: Why some like it radical. In Davis, R.B., Maher, C.A. & Noddings, N. (Eds.) *Constructivist Views on the Teaching and Learning of Mathematics*, pp. 19-30. Journal for Research in Mathematics Education, Monograph No. 4. Reston, Va.: NCTM.

von Glasersfeld, E. (1991) Radical Constructivism in Mathematics Education. Dordrect, The Netherlands: Kluwer.

von Glasersfeld, E. (1995) A Constructivist Approach to Teaching. In Steffe. L.P. & Gale, J. *Constructivism in Education.* Hillsdale, N.J.: Lawrence Erlbaum

Vygotsky, L.S. (1978) *Mind in Society* (Cole, M. et al., Eds.). Cambridge, Mass.: Harvard University Press.

Wertsch, J.V. (1985) *Vygotsky and the Social Formation of Mind.* Cambridge: Harvard University Press.

Wheatley, G. (1992) Reflection in Mathematics Learning and the Influence of Others. Paper presented at the Seventh International Congress on Mathematics Education, Quebec, Canada.

Whitehead, J. (1993) *The Growth of Educational Knowledge: Creating your own educational theories.* Bournemouth, Dorset: Hyde Publications.

Whitehead, J. (1999) *How Do I Improve my Practice? Creating a Discipline of Education Through Educational Enquiry.* Unpublished Ph.D. Dissertation, University of Bath.

Whitehead, J. (2000) How Do I Improve My Practice? Creating and legitimating an epistemology of practice. *Reflective Practice*, Vol. 1, No. 1, pp. 91-104.

Whitehead, J. (2003) What is educational in what I do for myself, for others and for the education of social foramtions? A contribution to a conversation on theory in action research. University of Limerick, 5-7th June.

Wilson, S.M. (November, 1995) Not Tension but Intention: A response to Wong's analysis of the researcher/teacher. *Educational Researcher*, pp. 19-21.

Winter, R. (1996) Some Principles and Procedures for the Conduct of Action Research. In Zuber-Skerritt, O. (Ed.) *New Directions in Action Research*, pp. 13-17. London: Falmer.

Wood, E. & Geddis, A.N. (1999) Self-conscious narrative and teacher education: representing practice in professional course work. *Teaching and Teacher Education*, Vol. 15, pp. 107-119.

Wong, E.D. (April, 1995) Challenges Confronting the Researcher/Teacher: Conflicts of purpose and conduct. *Educational Researcher*, pp. 22-28.

Wubbels, T. (1992) Taking Account of Student Teachers' Preconceptions. *Teaching and Teacher Education*, Vol. 8, No. 2, pp. 137-149.

Yackel, E. (1997) Explanation as an Interactive Accomplishment: A case study of one second-grade mathematics classroom. Paper prepared for the 1997 Annual Meeting of the American Educational Research Association, Chicago, Ill.

Yackel, E. et al. (1990) The Importance of Social Interaction in Children's Construction of Mathematical Knowledge. In Cooney, T.J. & Hirsch, C.R., *Teaching and Learning Mathematics in the 1990's*, pp. 12-21. 1990 Yearbook of the National Council of Teachers of Mathematics. Reston, Va: NCTM.

Yackel, E. & Cobb, P. (1996) Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education*, Vol. 27, No. 4, pp. 58-476.

Yinger, R.Y. (1987) Learning the Language of Practice. *Curriculum Inquiry* , Vol. 17 No. 3, pp. 293-318.

Zeichner, K.M. (1980) Myths and Realities: Field-based experiences in pre-service teacher education. *Journal of Teacher Education*, Vol. 31, pp. 45-55.

Zeichner, K.M. & Gore, J..M. (1990) Teacher Socialization. In Houston, W.R. (Ed.), *Handbook of Research on Teacher Education*, pp. 329-348. New York: Macmillan.

Zeichner, K.M. & Noffke, S. (199?) Practitioner Research as a Trustworthy Way of Knowing. Unpublished paper. University of Wisconsin - Madison.

Zeichner, K.M. & Tabachnik, B.R. (1985) The Development of Teacher Perspectives: Social strategies and institutional control in the socialization of beginning teachers. *Journal of Education for Teaching*, Vol. 11 No. 1, January 1985, pp. 1-25.

Zuber-Skerritt, O. (1992) *Action Research in Higher Education: Examples and Reflections*. London: Kogan Page.

Appendix A

David Yellin Teachers College

Course Description - 1997-1998

Name of Course: Didactics of Mathematics for Early Childhood

Name of Teacher: Rachel Deitcher

Semesters: A & B

Hours: 2

Goals: a. Understanding children's mathematical thinking
b. Understanding the processes of mathematical problem solving
c. Developing of reflective thinking and a critical attitude towards mathematics teaching
d. Familiarization with constructivist theory
e. Teaching methods that encourage children's construction of their own understandings
f. Familiarization and use of mathematics manipulatives and games
g. Integration of alternative assessment procedures within daily mathematics work
h. Familiarization with Feuerstein's theory of Mediated Learning Experience

Course Content:

Module 1
a. The development of logico-mathematical thinking
b. The conceptual development of children between the ages of 4 and 7
c. Graphs and diagrams
d. MLE - the basic criteria

Module 2
e. The development of number sense in children
conservation of number; counting; estimation; the whole and its parts; addition and subtraction
f. The use of games
g. MLE - additional parameters

Module 3
h. Mathematical problem-solving
i. Pattern
j. Multiplication and Division
k. The base-ten system

Module 4
l. Comparison and measurement
m. Spatial perception
n. Two-dimensional and three-dimensional shapes

Instructional Methods:

a. Participation in experiential mathematical activity
b. Critical reflection on pedagogical issues - practical and theoretical
c. The use of MLE in the analysis of one's own mathematics teaching and that of teachers and fellow student teachers
d. Lectures on chosen topics in mathematics education

Requirements:

a. Active participation in lessons
b. Teaching mathematics lessons in field placement classes as part of the regular student teaching framework, and reflective reports on four of these lessons
c. Reading assigned texts and reflective reports on some of these

Appendix B

Pre-Project Questionnaire - June, 1997

The following is a questionnaire that relates to your experience with and opinions regarding mathematics. Its purpose is to help in the preparation of the course "Didactics of Early Childhood Mathematics". Signing your names on the questionnaire will help me to get to know you, but is not essential.

In the questions for which a scale from 1 to 5 is provided, 5 means "to a great extent" and 1 means "not at all".

1. I like mathematics. 1 2 3 4 5

2. In general, I liked mathematics classes in elementary school.

1 2 3 4 5

3. In general, I liked mathematics classes in high school.

1 2 3 4 5

4. I feel comfortable with the idea of teaching mathematics in school.

1 2 3 4 5

5. In the pre-school where I worked this year they dealt with mathematical concepts:

1 2 3 4 5

6. In the following, two sorts of teachers are presented. Read the two texts and then comment on the positive and negative points of each one.

A) The first teacher, Nitza, likes mathematics and enjoys teaching it. She begins her lessons with explanation given to the whole class together. She then invites a few children to the blackboard to solve a problem while the others are watching, and asks the class whether they agree that the solution is correct. Finally she has the children work individually in their workbooks, each at their own speed.

B) The second teacher, Gila, has the children sit in groups according to the subject that they choose to study on that day. There are various manipulatives put out at each table. While working on the tasks the children consult with each other. Towards the end of the lesson Gila asks the different groups to report on the work they have done.

Your Comments:_____

_____

_____

_____

7. What expectations or requests do you have regarding the course "Didactics

of Mathematics for Early Childhood"?_____

_____

_____

Appendix C

Beginning-of-Year Questionnaire - November, 1997

This questionnaire is meant to supply me with information about your field placements in the schools - both the objective and the subjective conditions.

Field placement: Name of School_____ Grade_____
Teacher's Name_____

1.Describe in brief a typical mathematics lesson in your class.

_____

_____

_____

2. To what extent are the mathematics lessons in your class in accordance with your expectations? 1 2 3 4 5
low high

Please Explain_____

_____

4. What aspects of the lessons have made a particular impression on you?

_____

_____

5. To what extent do the children show interest in the lessons?
1 2 3 4 5
low high

6. Circle the portion of children that:

a. raise their hands	many	some	few
b. seem to be listening	many	some	few
c. actively participate	many	some	few
d. ask questions	many	some	few
e. seem to be excited about the subject	many	some	few

7. Do you take an active part in the lesson? Yes / No

Please explain_____

_____

8. Are you satisfied with your role? Yes / No

Please explain_____

_____

Appendix D

Feedback on First Semester

1. What is the most important thing that you have learned since the beginning of the year?

_____

_____

_____

2. Note two activities that made the greatest impression on you.

_____

_____

_____

3. Have you changed your mind regarding any issues since the beginning of the year? If so, what was the most significant change?

_____

_____

_____

4. What ideas that you learned in the course have you put into practice with children?

_____

_____

_____

5. What changes would like to see in the way the course is run?

_____

_____

_____

Comments_____

_____

_____

Appendix E

End-of-Year Feedback

1. Have there been any changes in your perceptions of mathematics education during the course of the year? If so, what were they?

2. Do you feel that the learning strategies you experienced in this course will be appropriate for your work with children? Explain.

3. Note three educational principles that you considered during the course that seem to you most important and that you are determined not to give up on in your work with children.

4. Do you have any suggestions for improving the course in the future?

Appendix F

Instructions for Lesson Reporter

Whole-Class Work

I am interested mainly in a report of the interactions between myself and the students. What is most important to me at this stage is the way in which I provide mediated learning experiences - where did I mediate, what kind of mediation did I use, what in your opinion was its effect?

In addition to this I am interested in knowing how I accept and react to issues that you raise. Do I allow you to express yourselves, do I really listen to what you are saying, do I take advantage of your comments during the lesson, a how do I react when you don't understand something? The details of the content that we are working on in the lesson interest me only to the extent that they are relevant to our interactions.

Small-Group Work

Here I would like you to concentrate on the interactions between the students. Do the participants mediate for each other, who initiates the activity, who takes responsibility for the continuation of the work, is everyone participating, what roles do the different students take (who keeps notes, who reports on the work, who organizes the materials, who represents the group if there are questions).